



Tomography in the Ecliptic using HI-1

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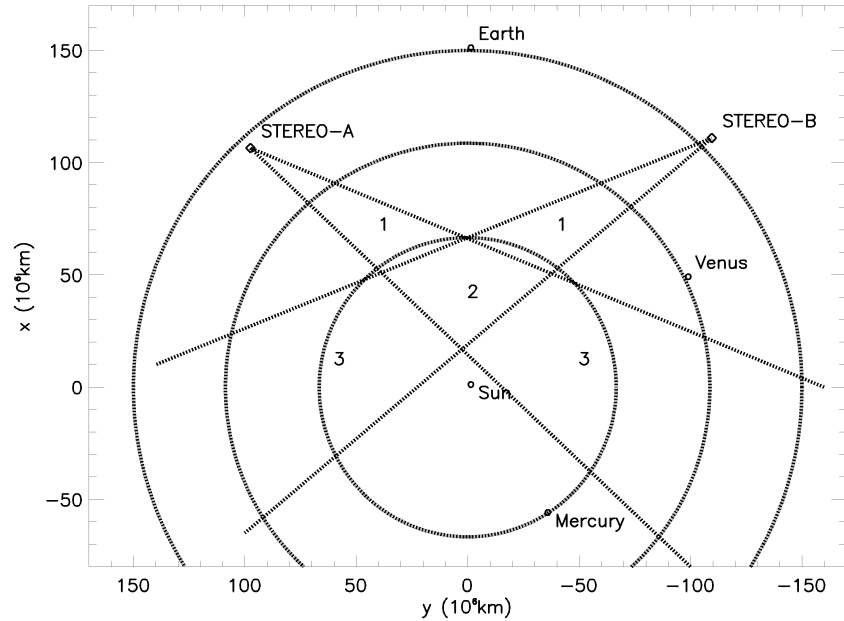
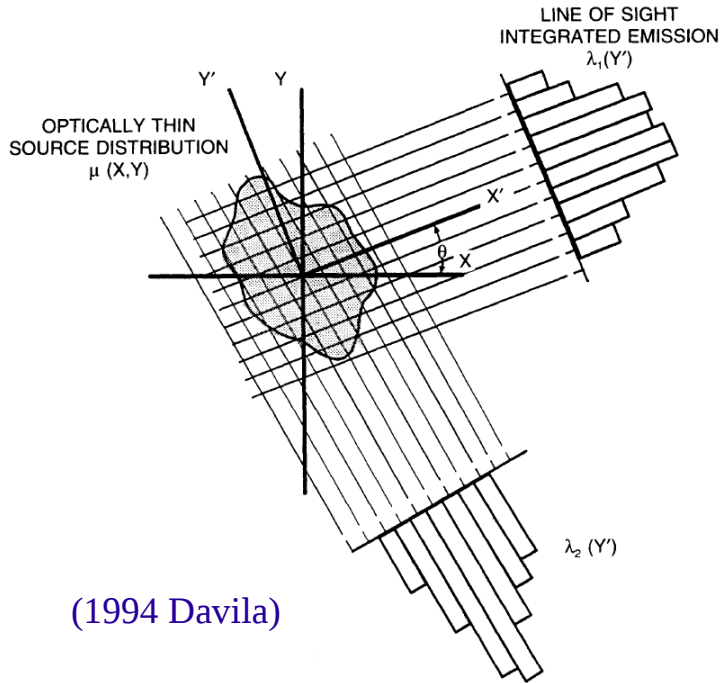
²*University College London*

Overview

- Theory of tomographic inversion applied to electron densities in the heliosphere
- Using HI data to formulate an inverse equation based on Thomson scattering by electrons
- Solution to the inverse equation using iterative estimation
- Separating the F- and K-corona in HI data
- Regularisation of the inverse equation (eg. positive densities, smooth gradient)
- Example of the method applied to Earth directed CMEs

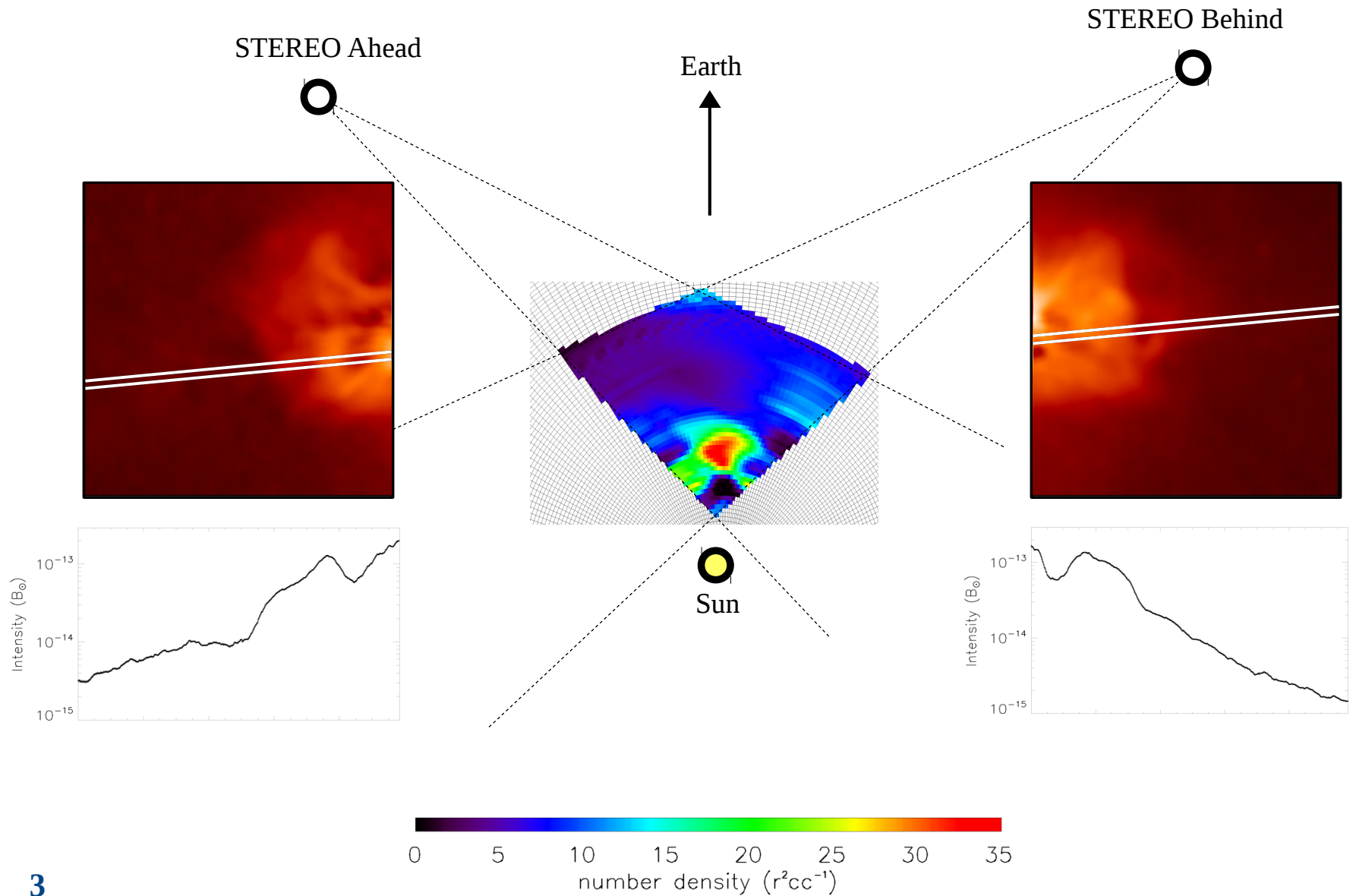


Tomography: Theory

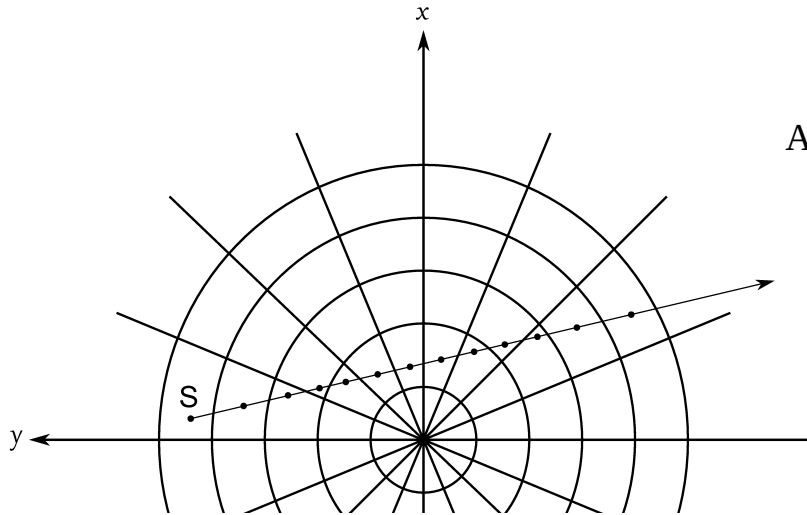


- Single observing position produces line-of-sight ambiguities
- These may be avoided using multiple points of view
- Tomography may be performed in the region common to FOV of STEREO A and B

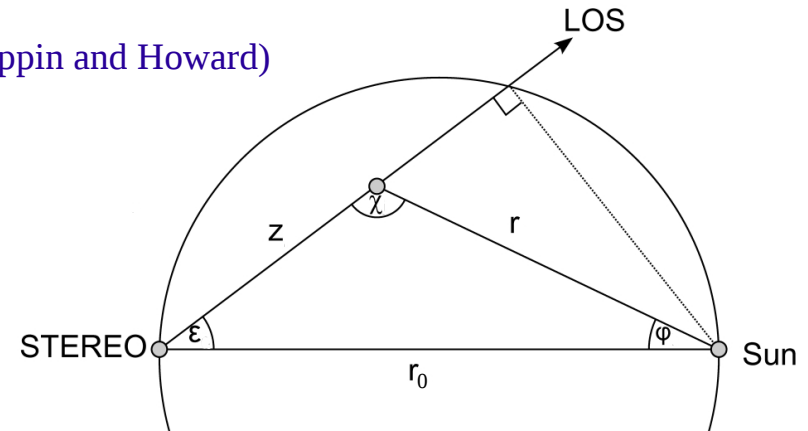
Tomography with STEREO HI



The Inverse Equation



A(2009 Tappin and Howard)



- The equation $\mathbf{y} = \mathbf{H}\mathbf{x}$ may be formulated by applying a grid to the heliosphere, with a resolution is 1.5° longitudinally by $1/160$ AU radially
- \mathbf{y} = data array, \mathbf{H} = physical operator, \mathbf{x} = density array
- $\mathbf{H}(r, \chi)$ is based on Thomson scattering
- \mathbf{H} and \mathbf{y} are known, which means we can find \mathbf{x}

Solving the Inverse Problem

$$\mathbf{y} = \mathbf{H}\mathbf{x}$$

\mathbf{y} : M ~ 2000 elements

\mathbf{x} : N ~ 5000 elements

\mathbf{H} : NxM elements... $> 10^7$

\therefore Too big to solve Analytically

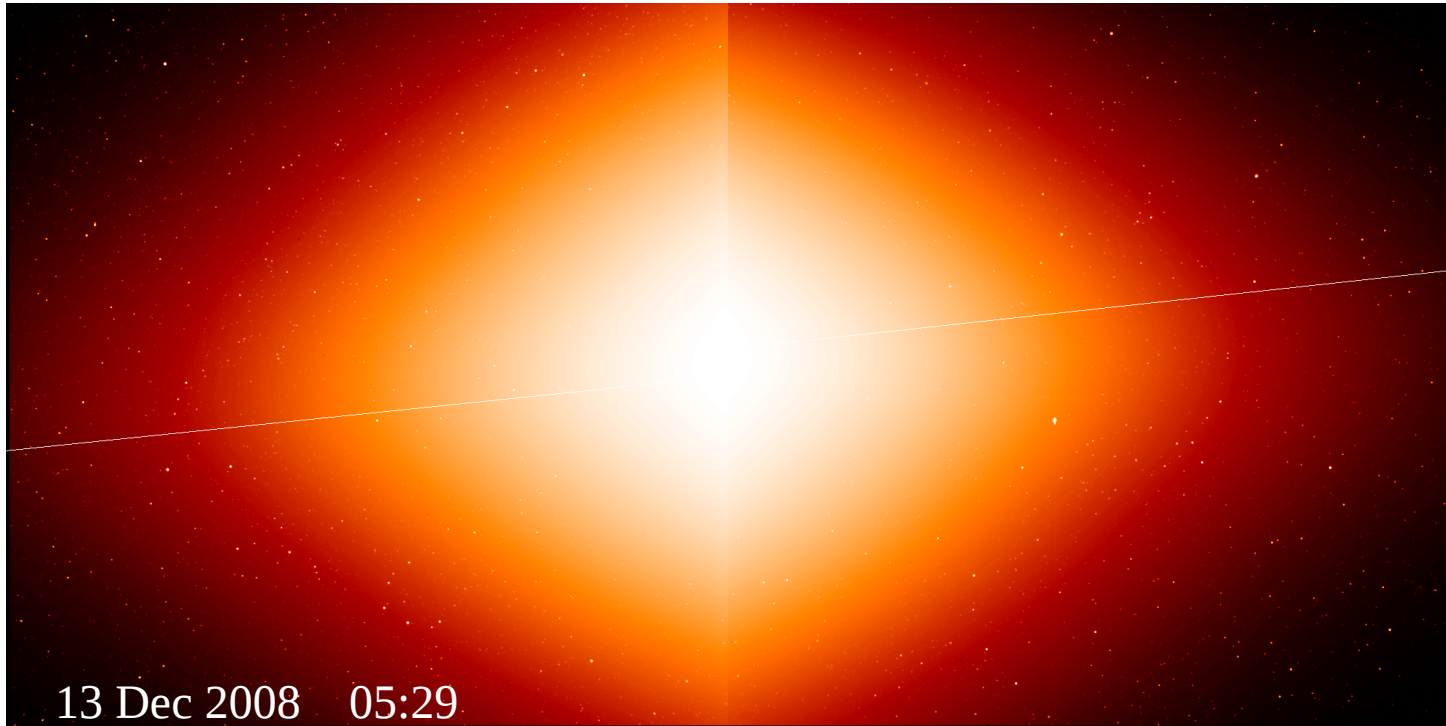
- Instead we solve it using an iterative estimation method: the *Conjugate gradient algorithm*
 - make initial guess at density array: $\mathbf{x}_o = 8r^{-2}$ electrons cc^{-1}
 - Find residual $\mathbf{r}_o = \mathbf{y} - \mathbf{H}\mathbf{x}_o$
 - update value of \mathbf{x}_i such that the value of \mathbf{r}_i is reduced
 - repeat until \mathbf{r}_i is sufficiently small



Background Subtraction

STEREO A

STEREO B

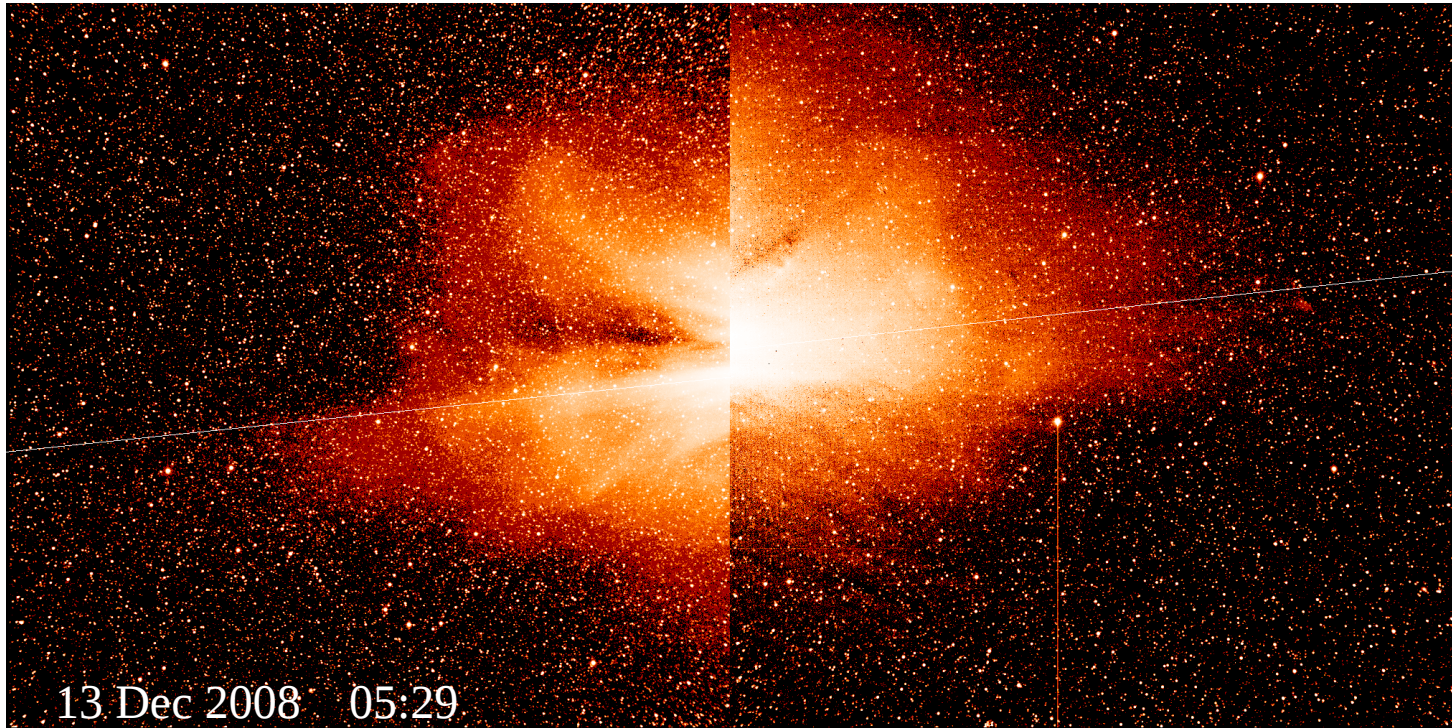


- HI-1 data dominated by F-corona (dust)

Background Subtraction

STEREO A

STEREO B

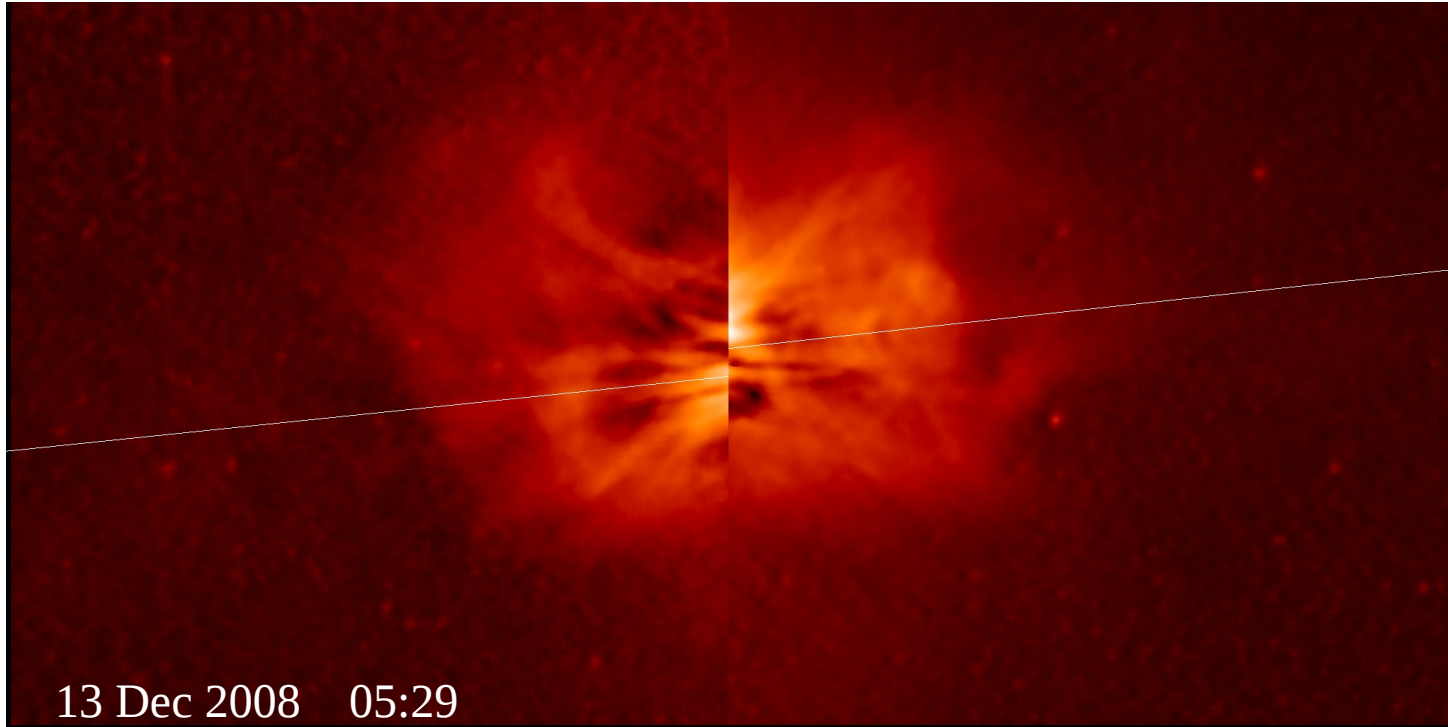


- HI-1 data dominated by F-corona (dust)
- One-day minimum subtracted from each pixel

Background Subtraction

STEREO A

STEREO B

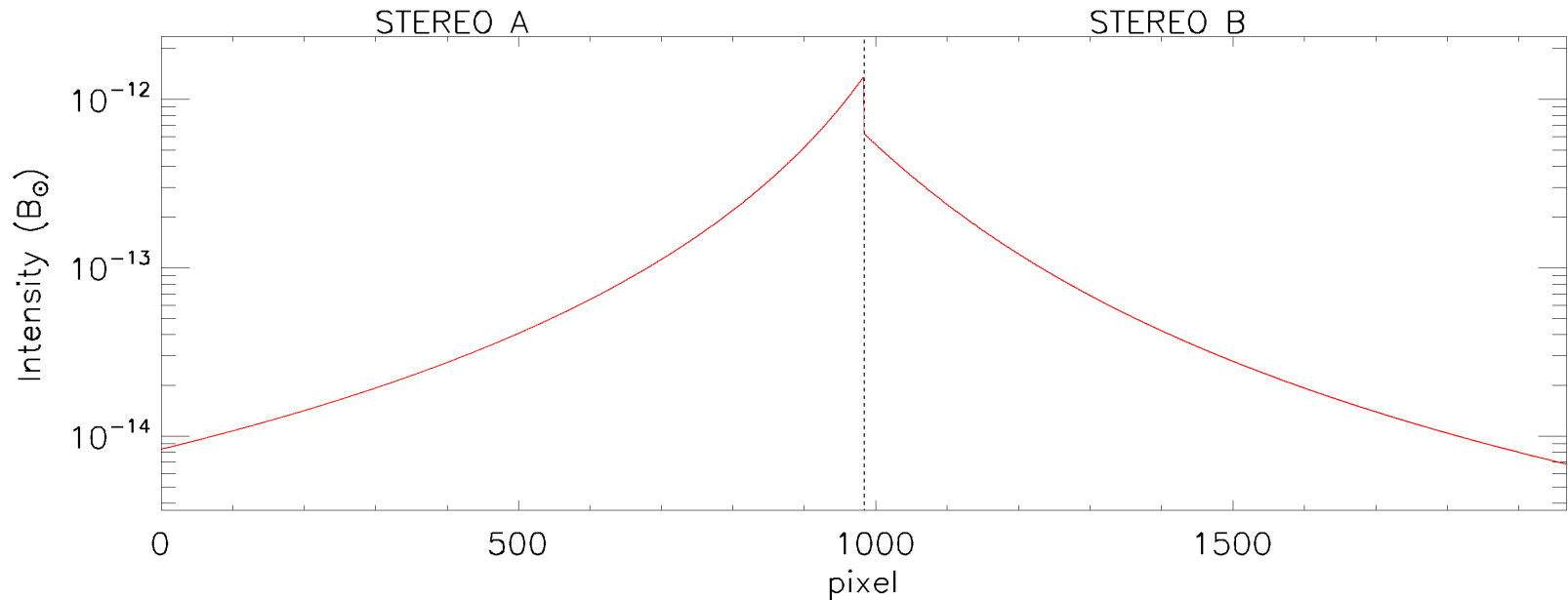


- HI-1 data dominated by F-corona (dust)
- One-day minimum subtracted from each pixel
- Apply median filter over nearby pixels



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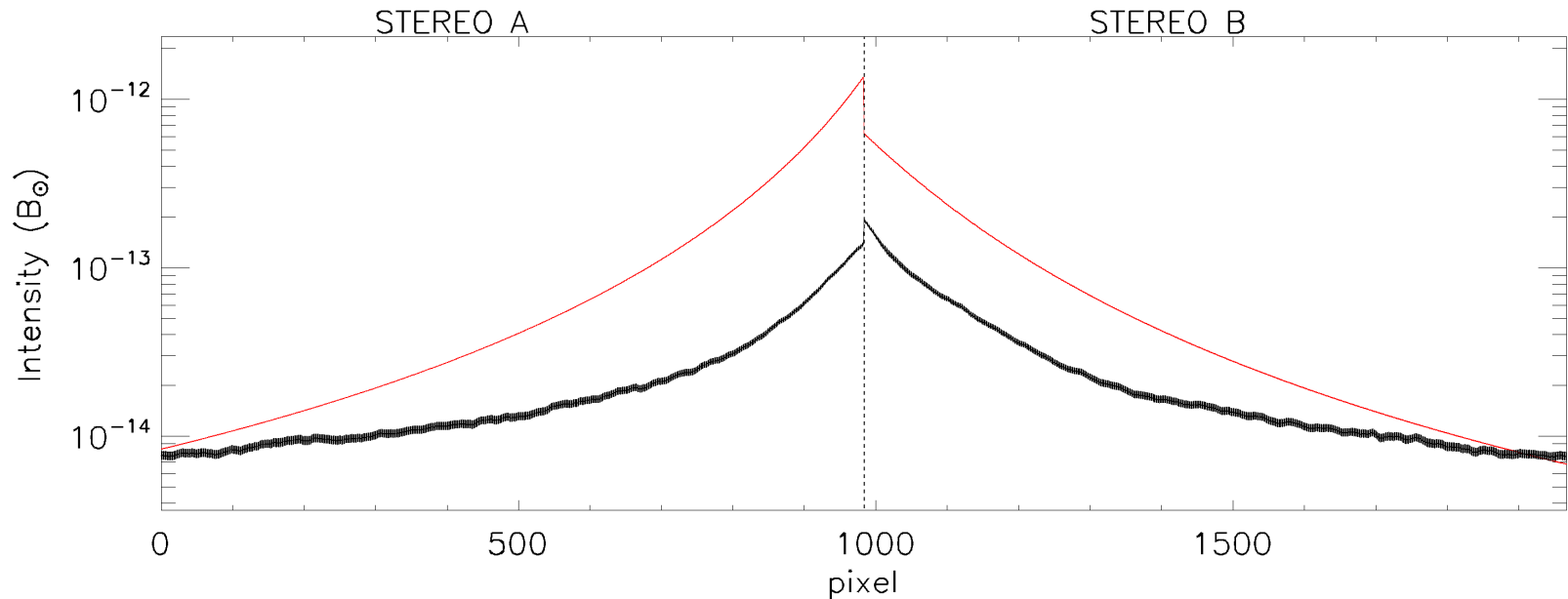
K-Corona Intensity Correction



- Use $y = Hx$ to determine theoretical values of intensity

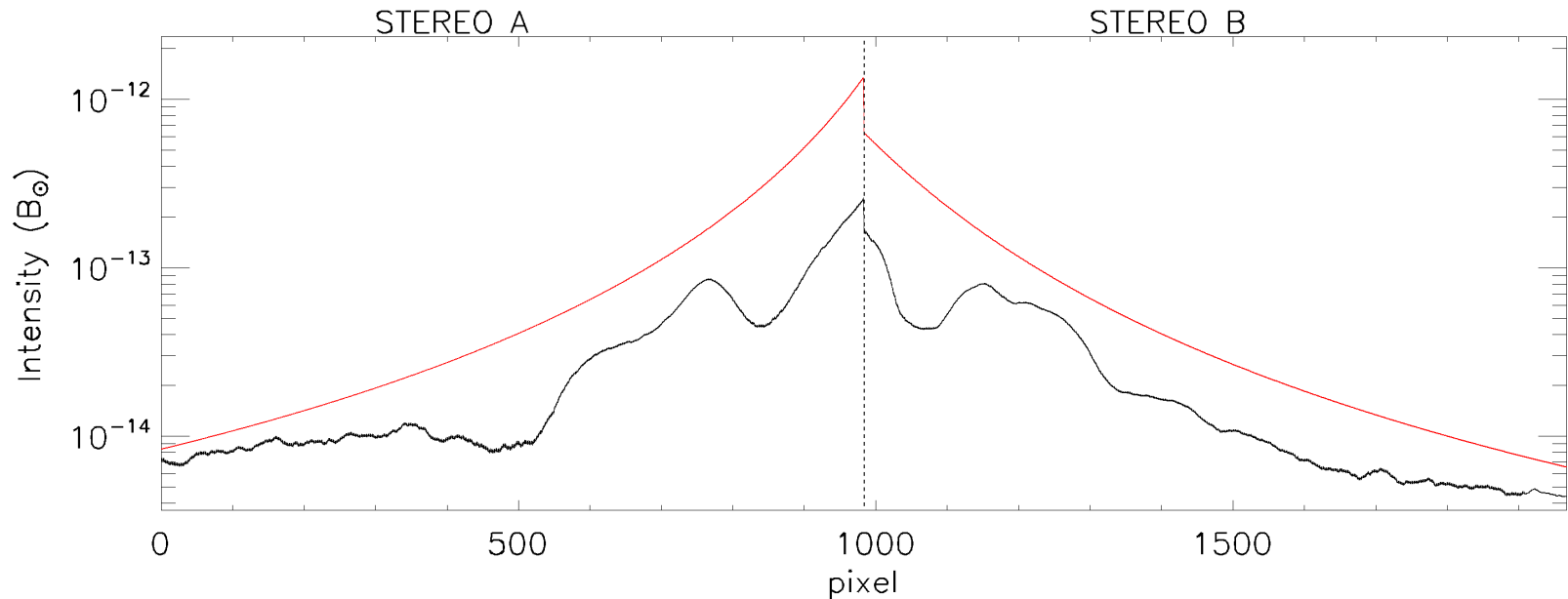
– Theoretical Intensity, $x = 8.0r^2$ electrons cc^{-1}

K-Corona Intensity Correction



- Use $y = Hx$ to determine theoretical values of intensity
- Ratio of *theoretical* to *observed* provides a correction factor
- Theoretical Intensity, $x = 8.0r^2$ electrons cc^{-1}
- Observed Intensity, two-week average (28 Nov – 11 Dec 2008)

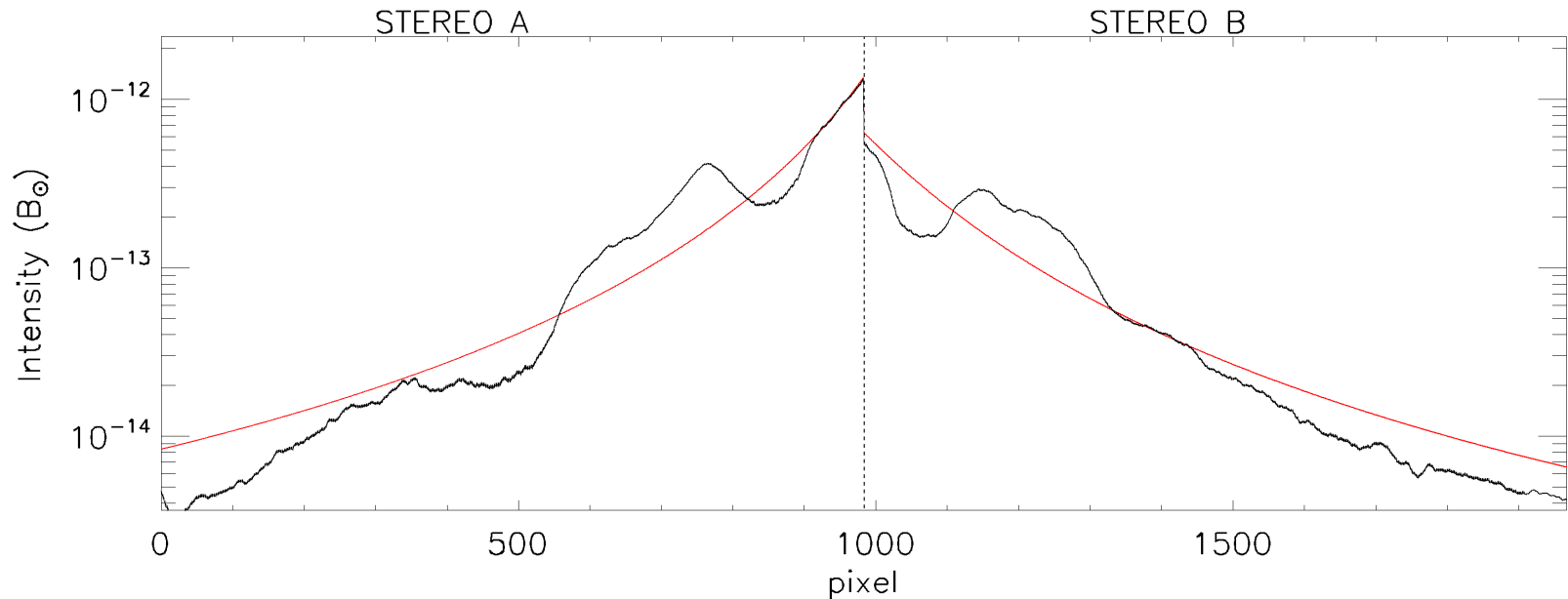
K-Corona Intensity Correction



- Use $y = Hx$ to determine theoretical values of intensity
- Ratio *theoretical* to *observed* provides a correction factor
- This may be applied to individual HI images

- Theoretical Intensity, $x = 8.0r^2$ electrons cc^{-1}
- Observed Intensity, 13 Dec 2008 05:29

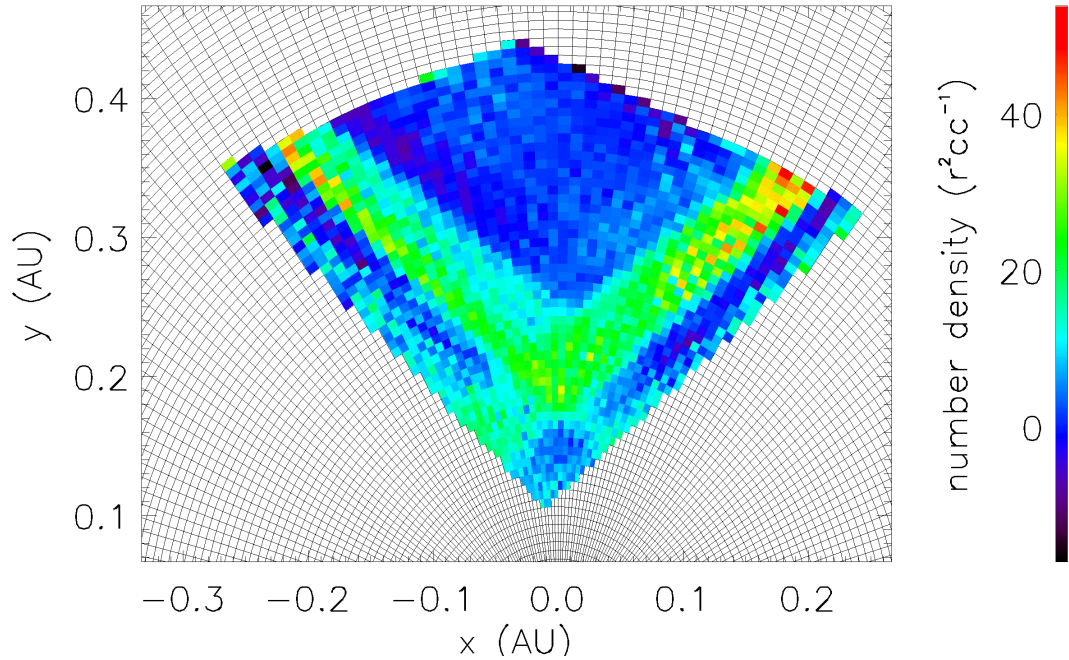
K-Corona Intensity Correction



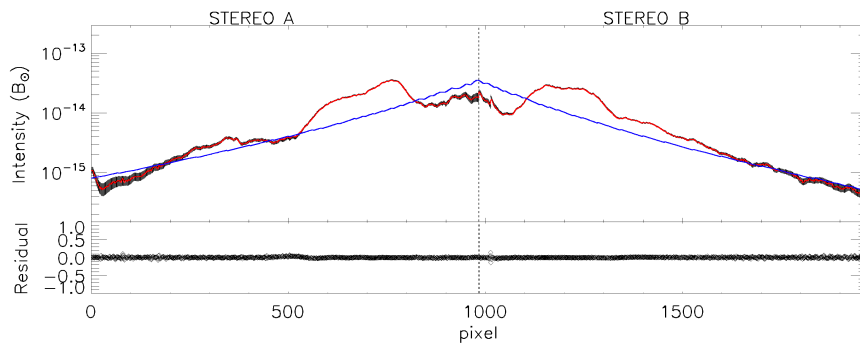
- Use $y = Hx$ to determine theoretical values of intensity
- Ratio *theoretical* to *observed* provides a correction factor
- This may be applied to individual HI images

- Theoretical Intensity, $x = 8.0r^2$ electrons cc^{-1}
- Corrected Intensity, 13 Dec 2008 05:29

Regularisation

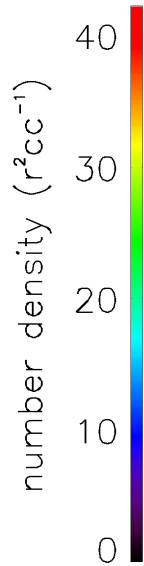
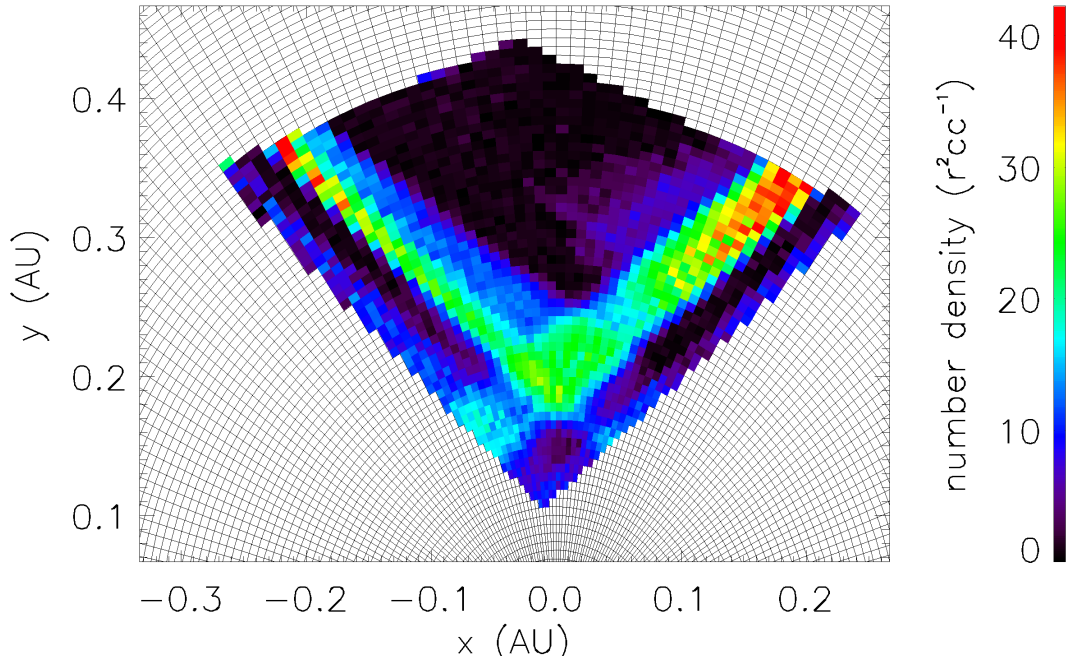


- 'Map' represents density array, x

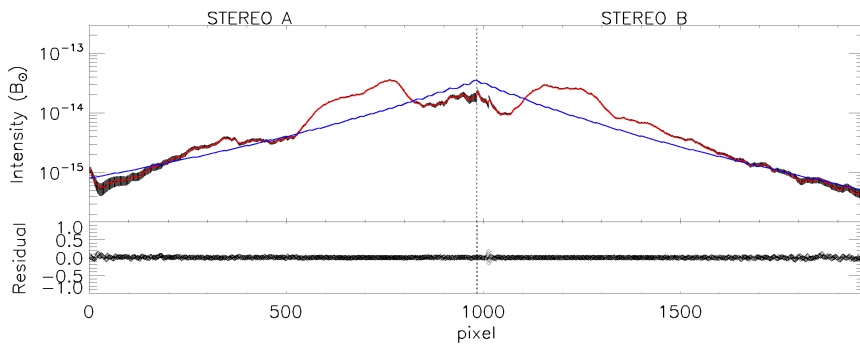


- Data
- Initial guess $x = 8.0r^2$
- Solution

Regularisation

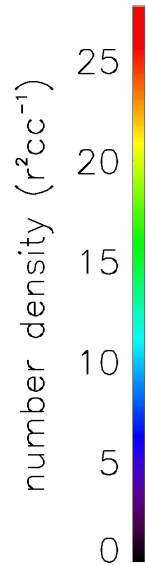
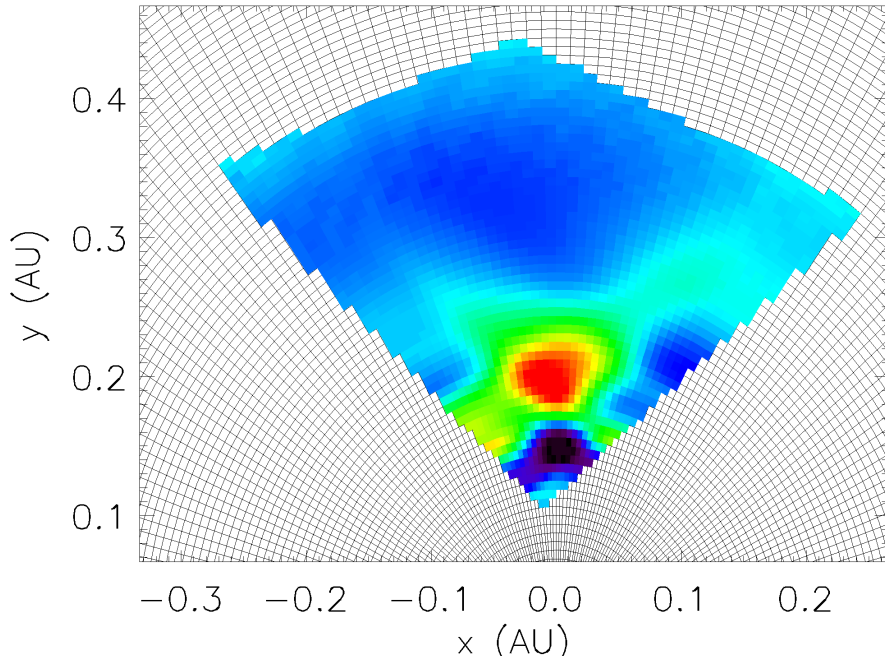


- 'Map' represents density array, x
- Reset negative values to x_0 after each iteration

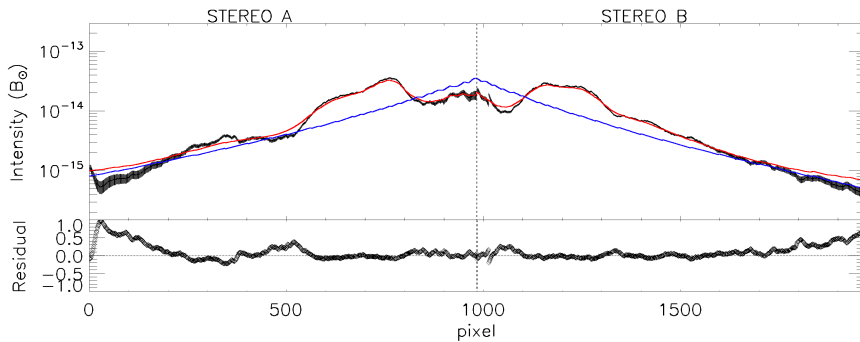


- Data
- Initial guess $x = 8.0r^2$
- Solution

Regularisation

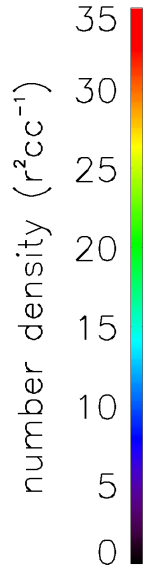
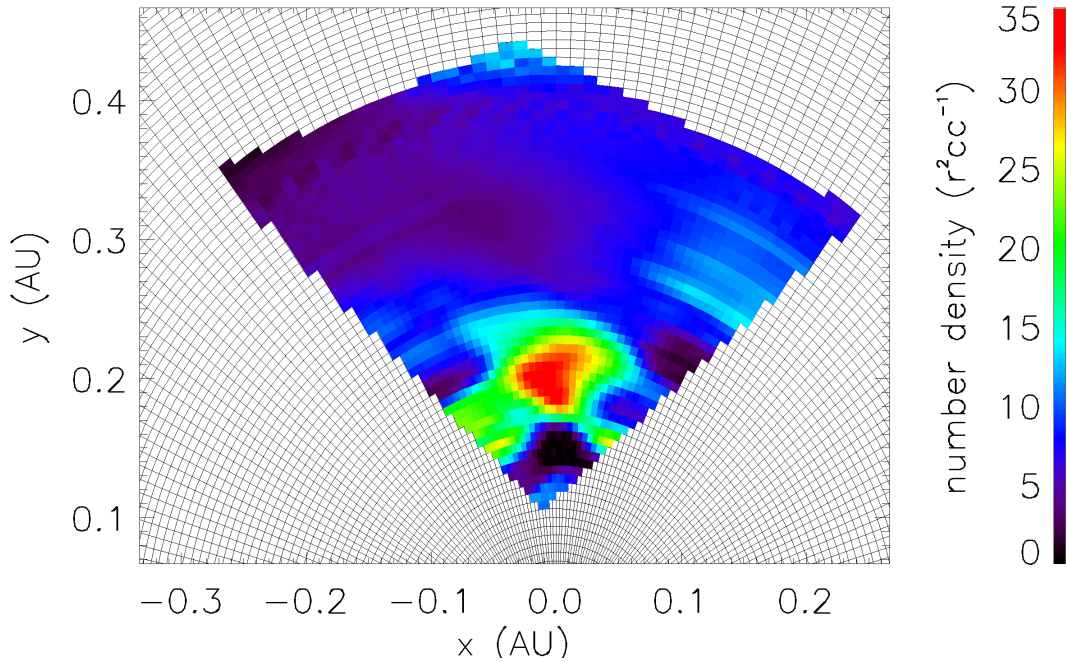


- 'Map' represents density array, \mathbf{x}
- Reset negative values to \mathbf{x}_0 after each iteration
- Include 'regularisation' matrices, $D\varphi$ and D^2r

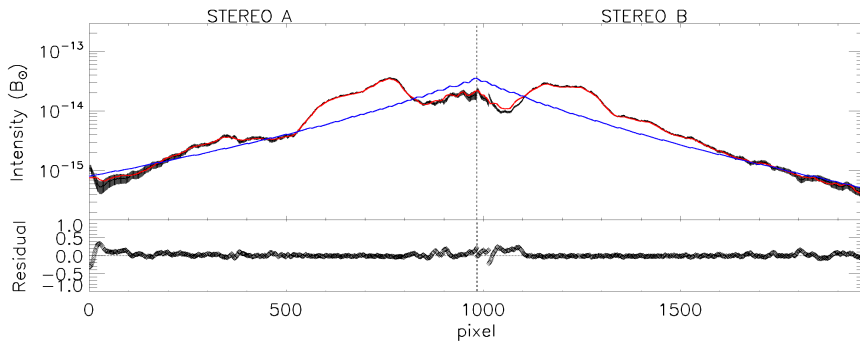


- Data
- Initial guess $x = 8.0r^2$
- Solution

Regularisation

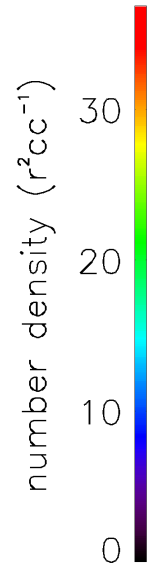
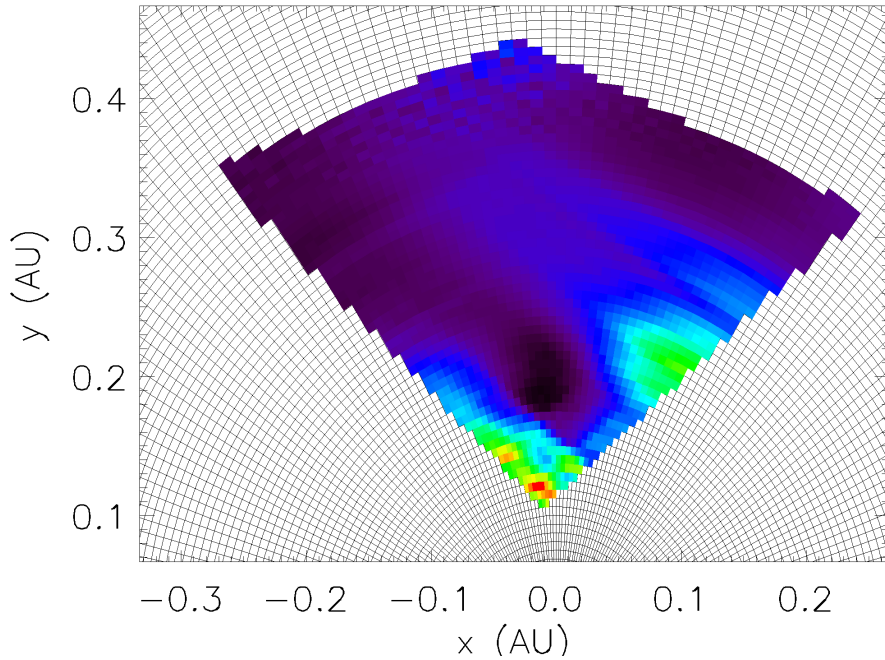


- 'Map' represents density array, \mathbf{x}
- Reset negative values to x_0 after each iteration
- Include 'regularisation' matrices, $D\varphi$ and D^2r
- Add weighting each pixel to account for range of intensities (factor $\sim 10^2$)



- Data
- Initial guess $x = 8.0r^2$
- Solution

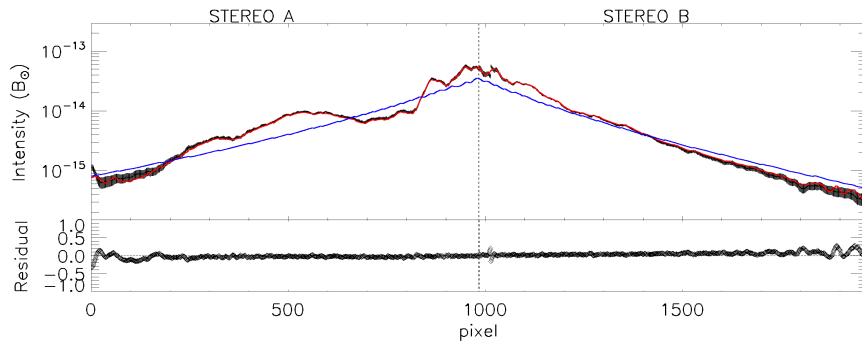
Density Results – Dec 2008 CME



12 Dec 2008 19:29

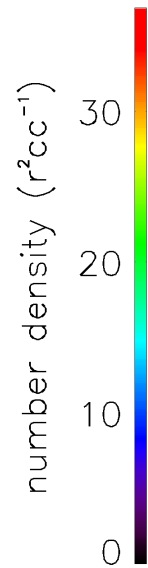
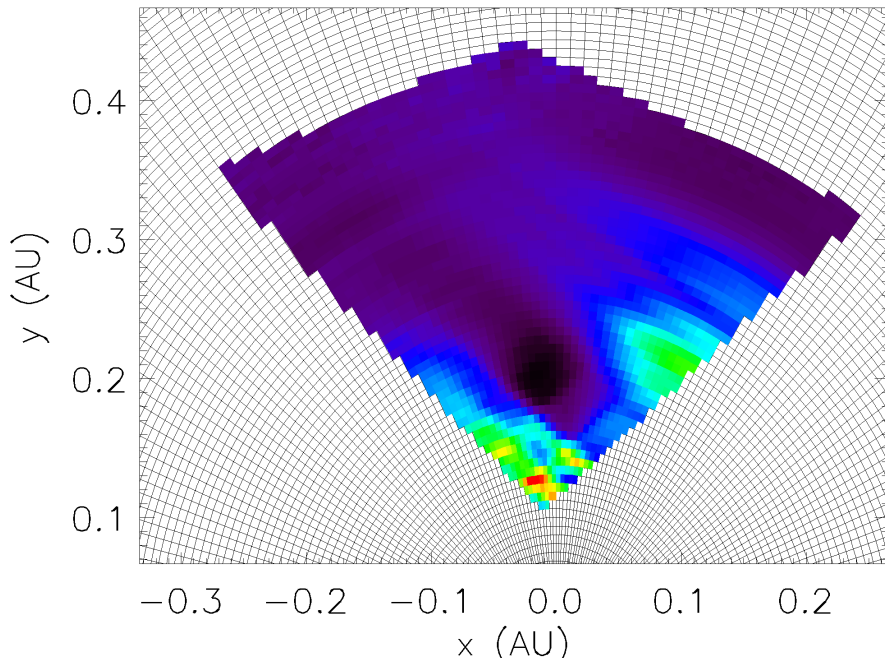
Density (peak) = $35.8 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $18 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

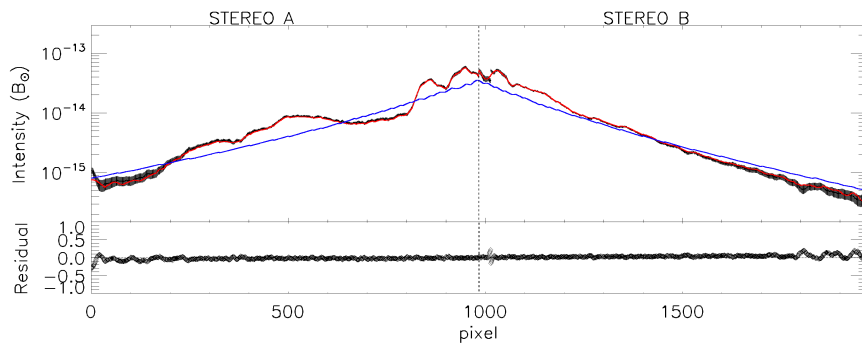
Density Results – Dec 2008 CME



12 Dec 2008 20:09

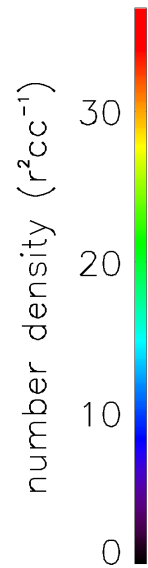
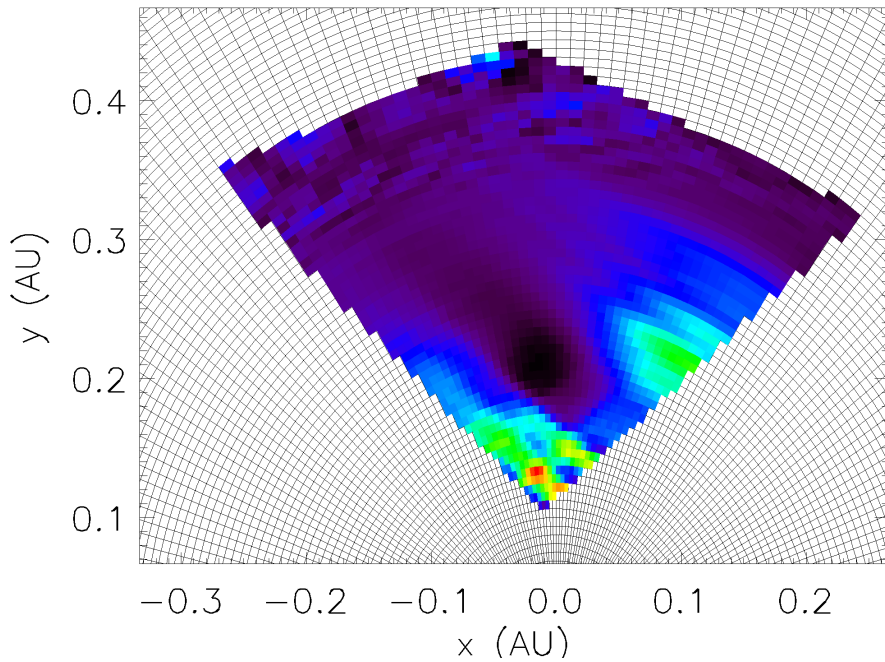
Density (peak) = $35.7 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $19 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

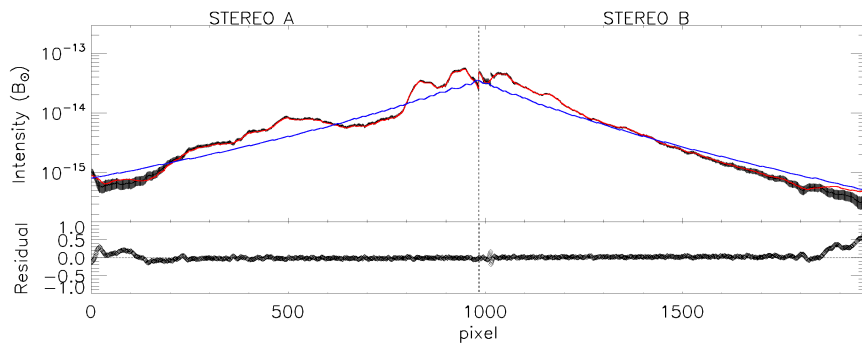
Density Results – Dec 2008 CME



12 Dec 2008 20:49

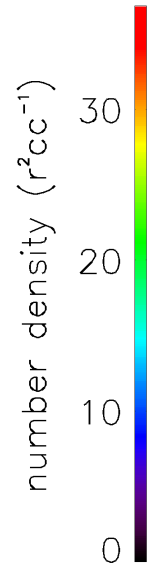
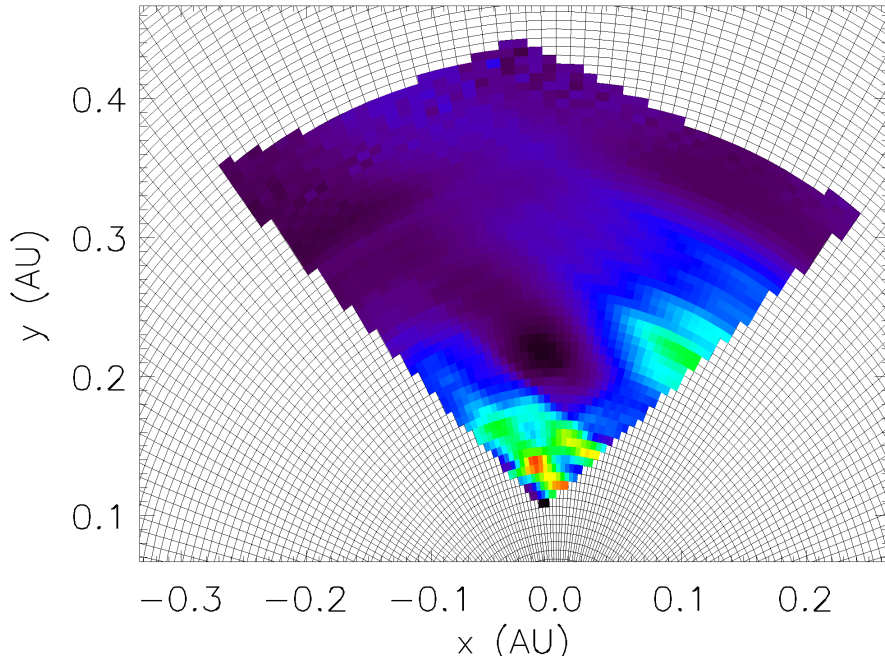
Density (peak) = $34.8 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $20 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

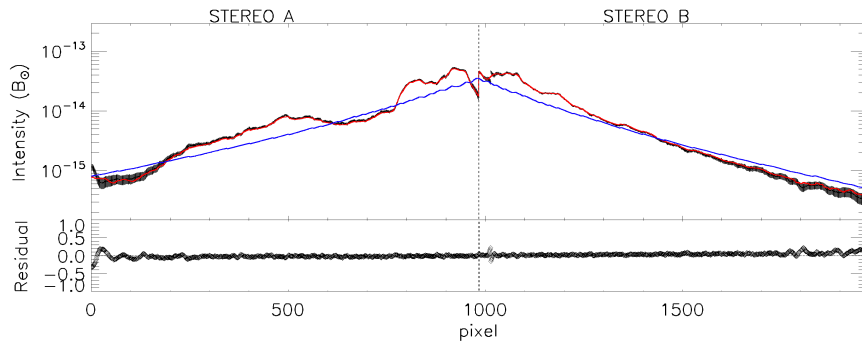
Density Results – Dec 2008 CME



12 Dec 2008 21:29

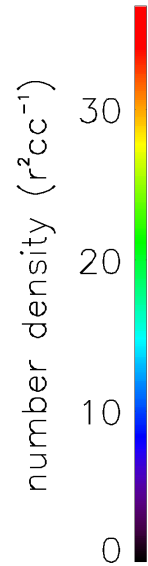
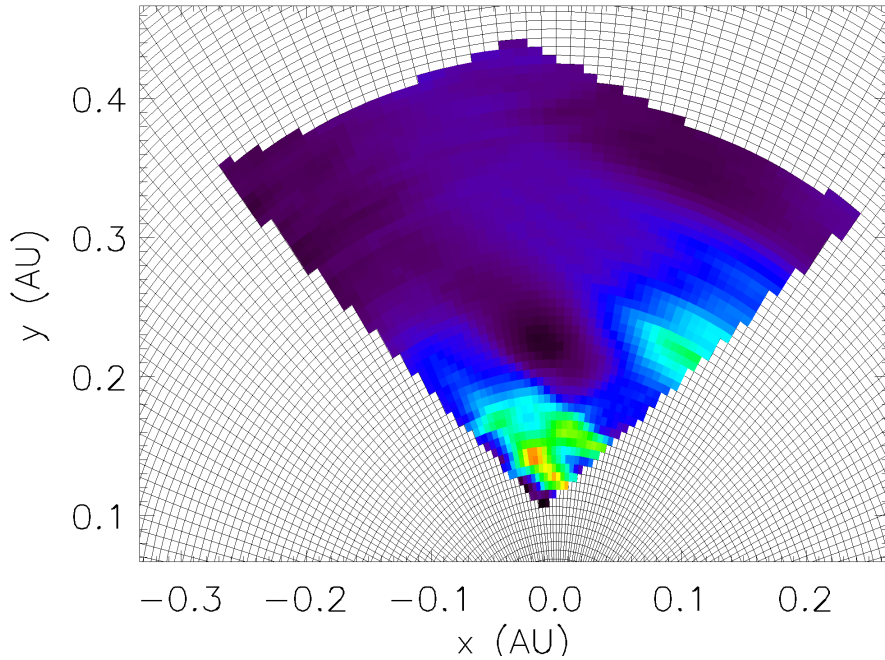
Density (peak) = $34.8 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $20 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

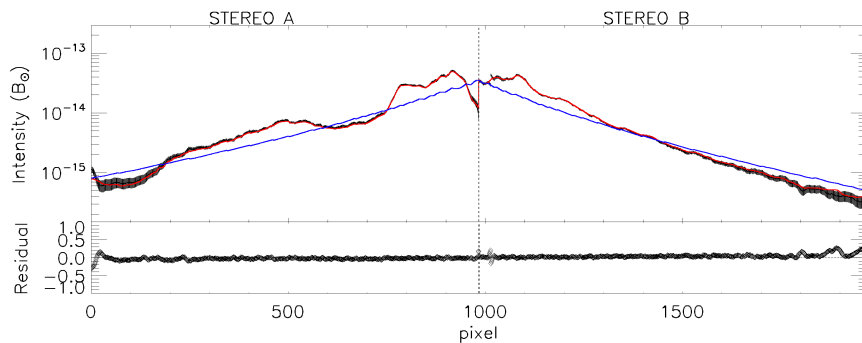
Density Results – Dec 2008 CME



12 Dec 2008 22:09

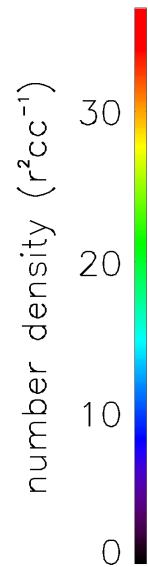
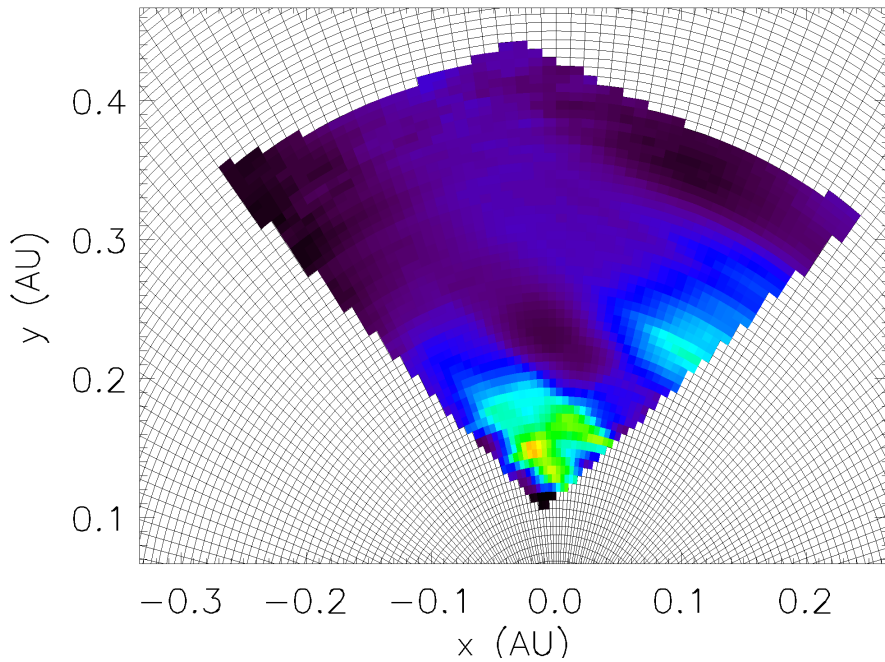
Density (peak) = $32.9 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $21 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

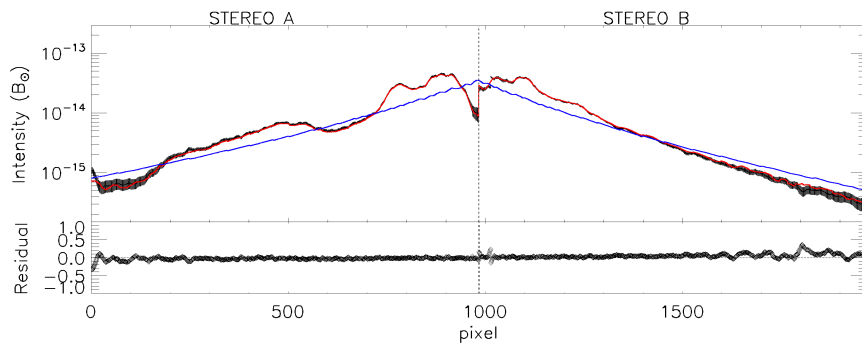
Density Results – Dec 2008 CME



12 Dec 2008 22:49

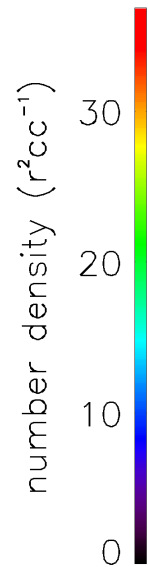
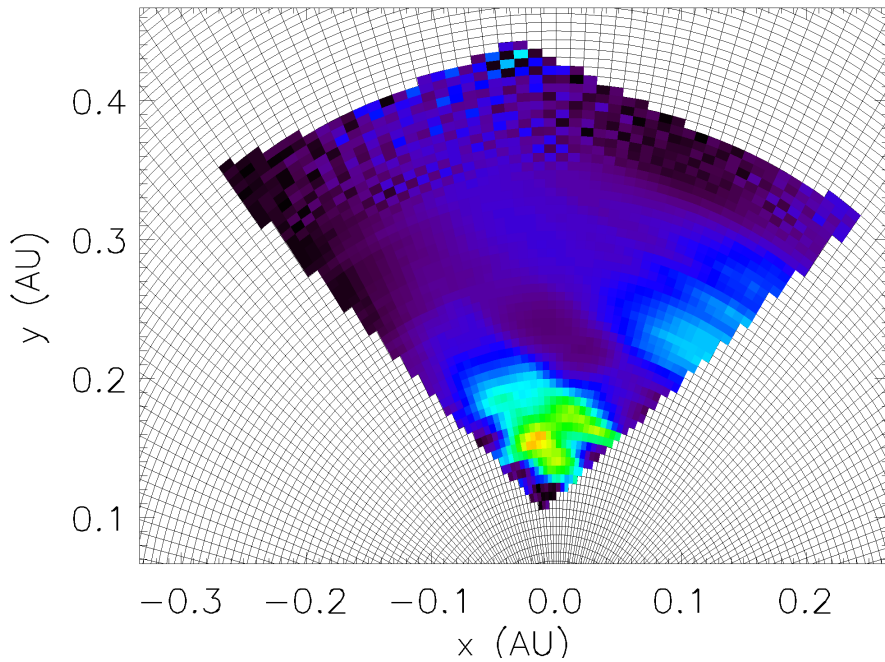
Density (peak) = $31.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $21 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

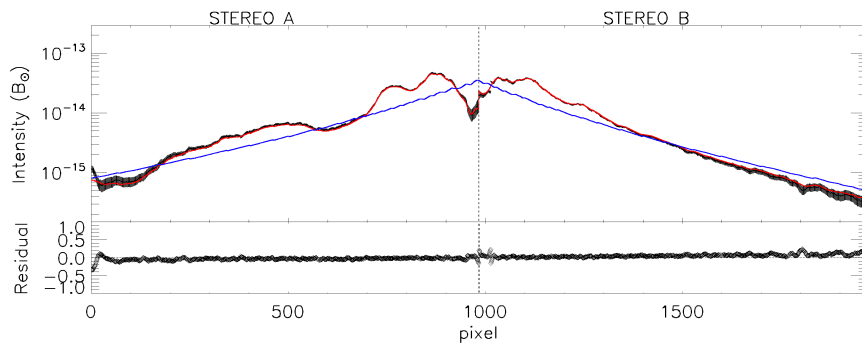
Density Results – Dec 2008 CME



12 Dec 2008 23:29

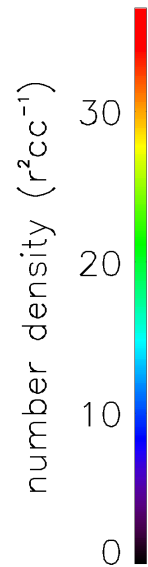
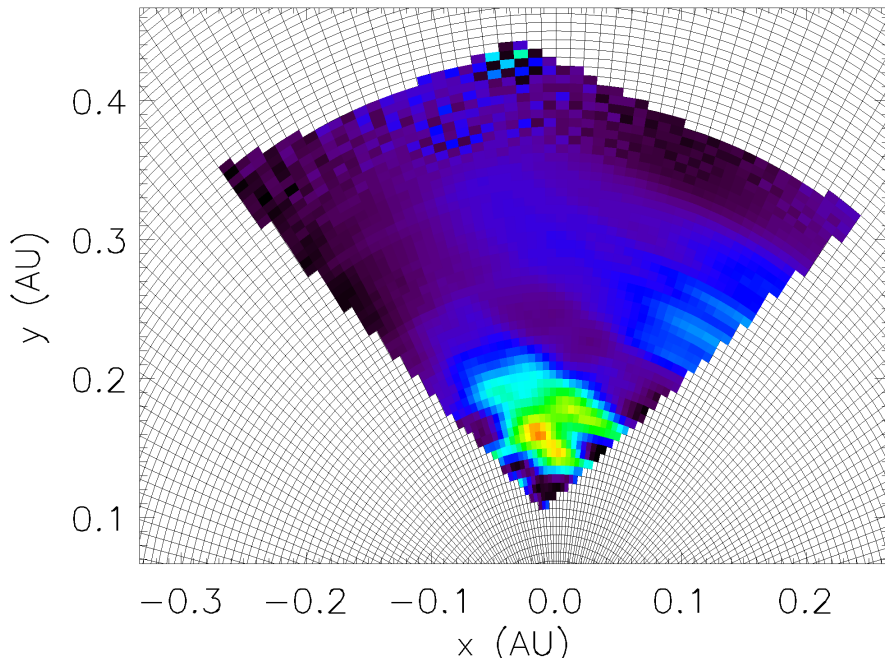
Density (peak) = $29.6 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $22 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

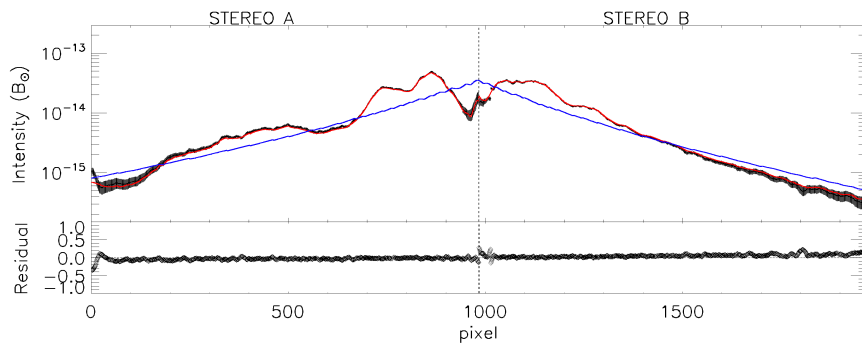
Density Results – Dec 2008 CME



13 Dec 2008 00:09

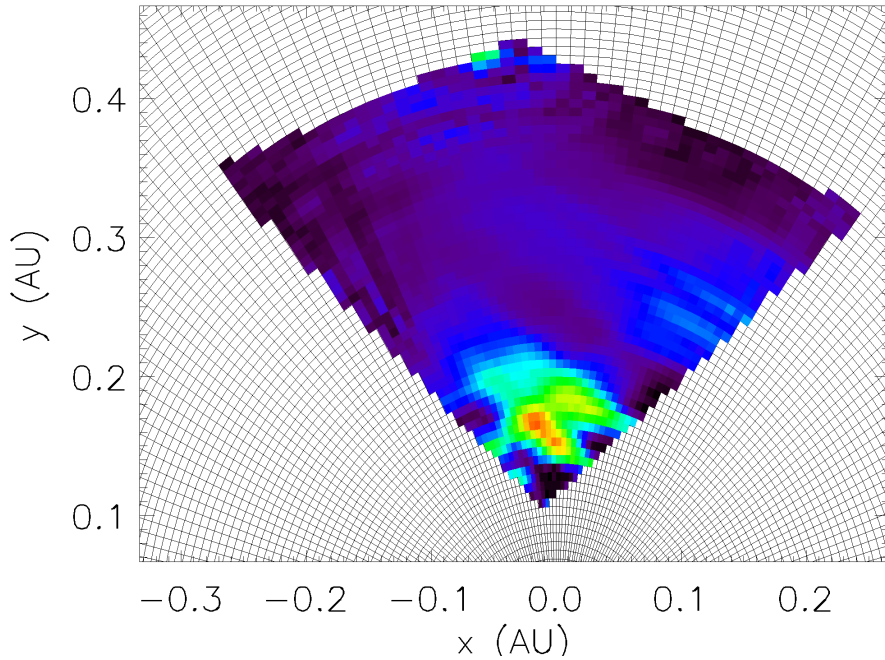
Density (peak) = $29.6 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $23 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

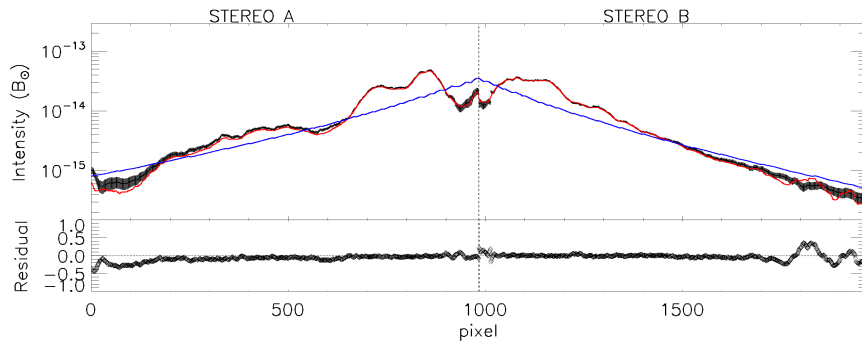
Density Results – Dec 2008 CME



13 Dec 2008 00:49

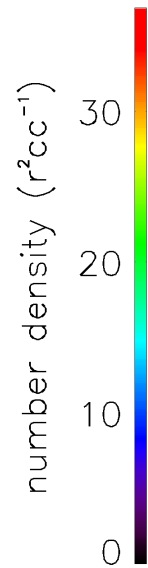
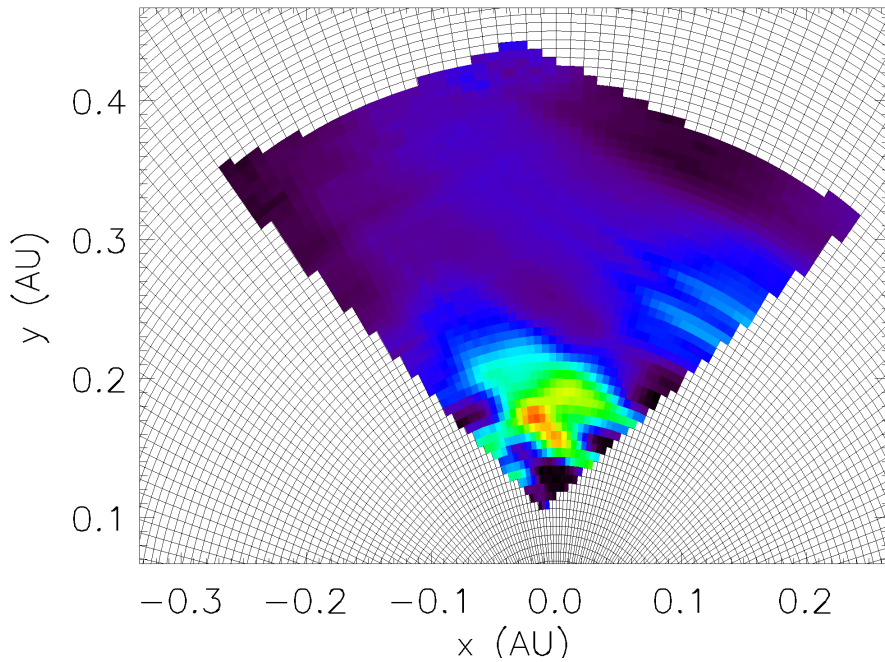
Density (peak) = $31.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $23 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

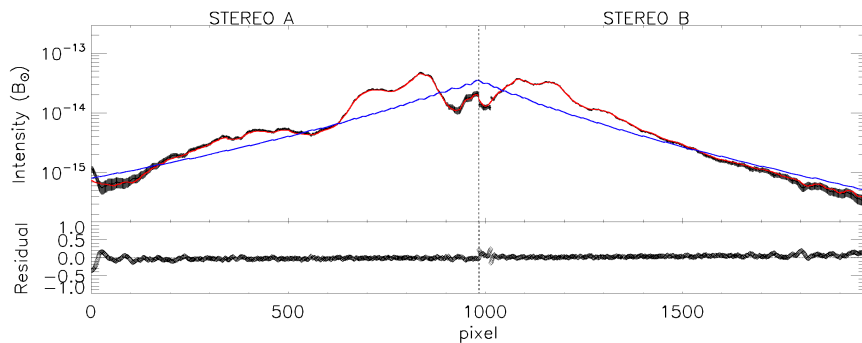
Density Results – Dec 2008 CME



13 Dec 2008 01:29

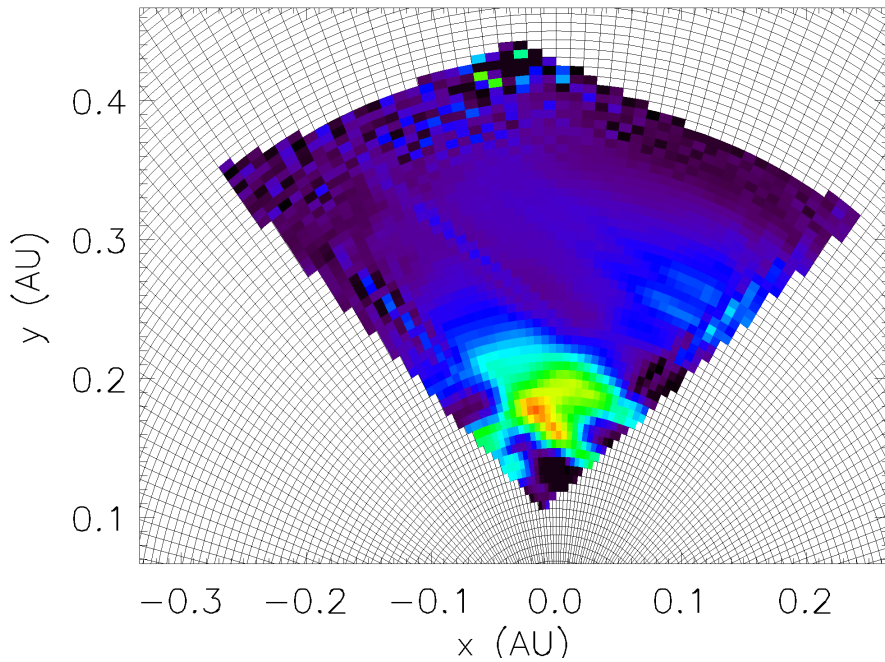
Density (peak) = $32.2 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $24 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

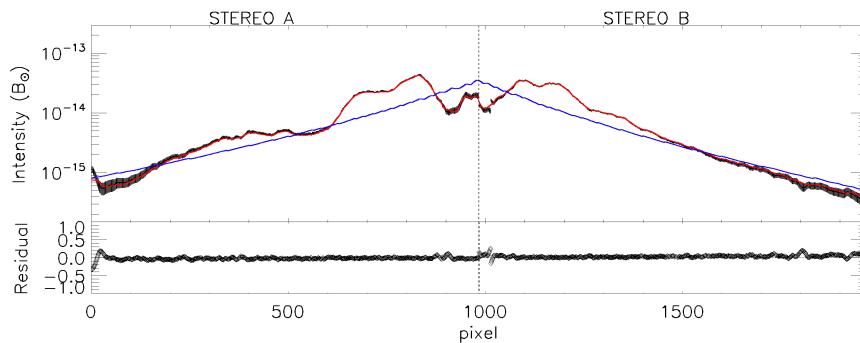
Density Results – Dec 2008 CME



13 Dec 2008 02:09

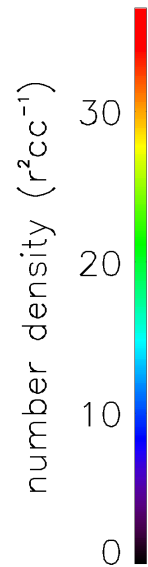
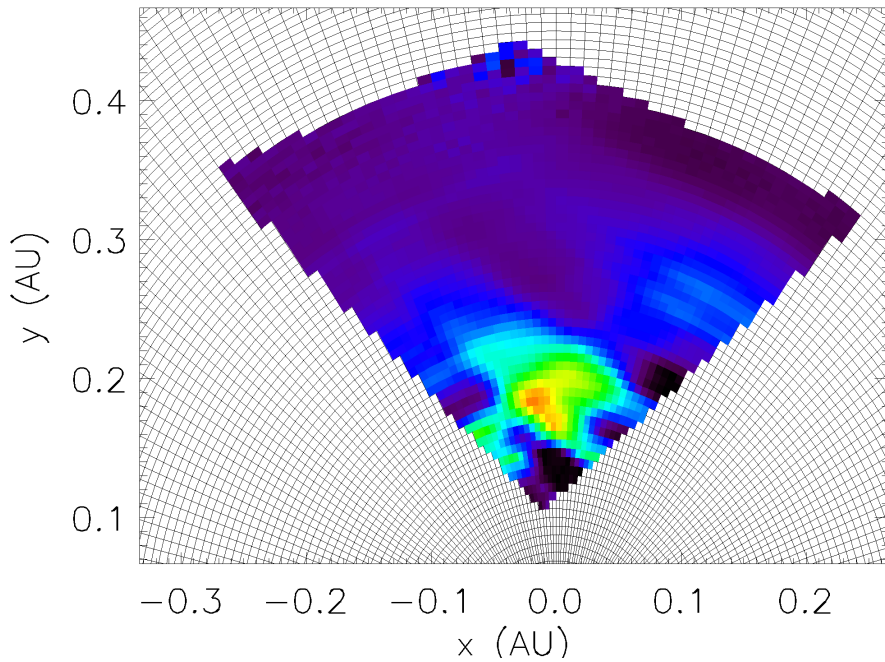
Density (peak) = 32.4 $e^- r^2 \text{cc}^{-1}$

Distance = 25 * 10^6 km



- Data
- Initial guess
- Solution

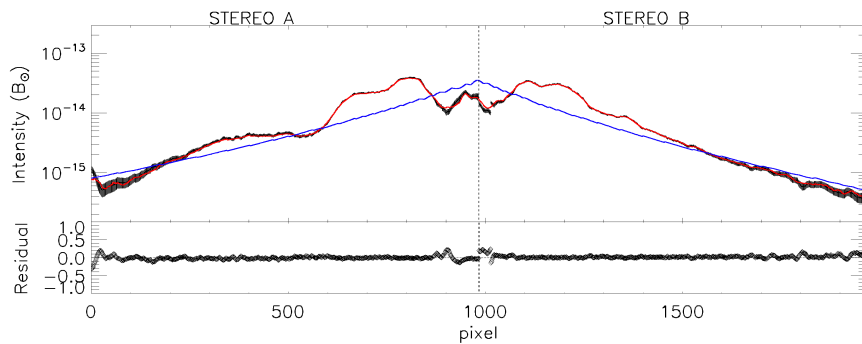
Density Results – Dec 2008 CME



13 Dec 2008 02:49

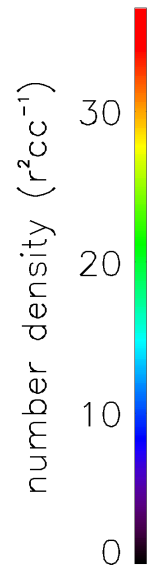
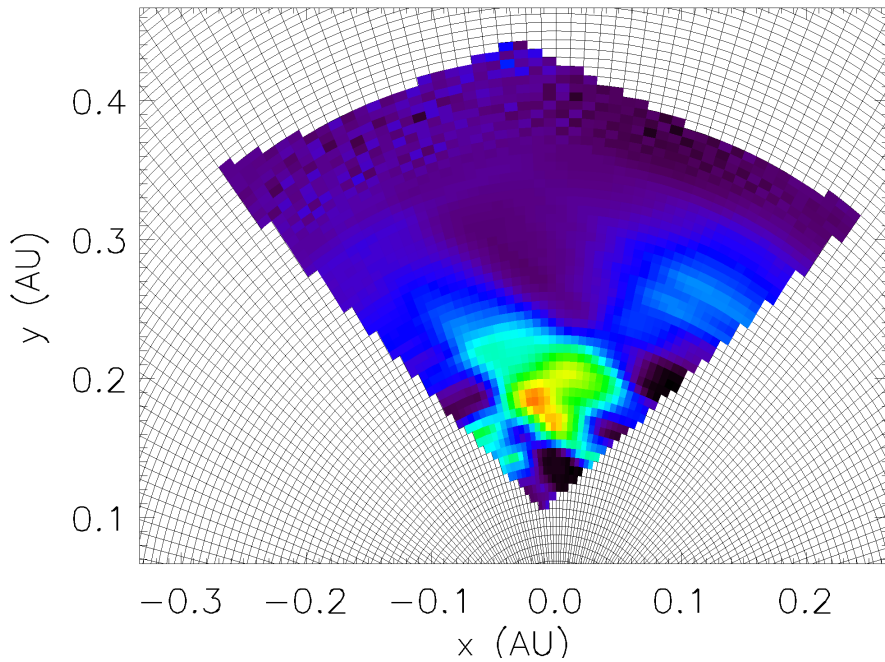
Density (peak) = $32.0 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $26 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

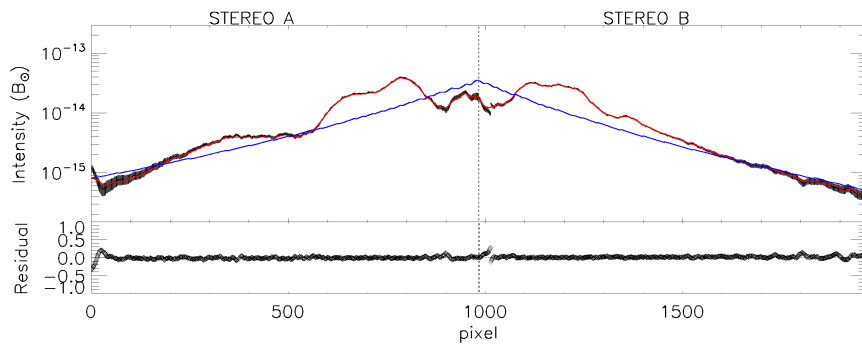
Density Results – Dec 2008 CME



13 Dec 2008 03:29

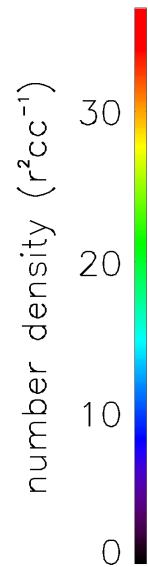
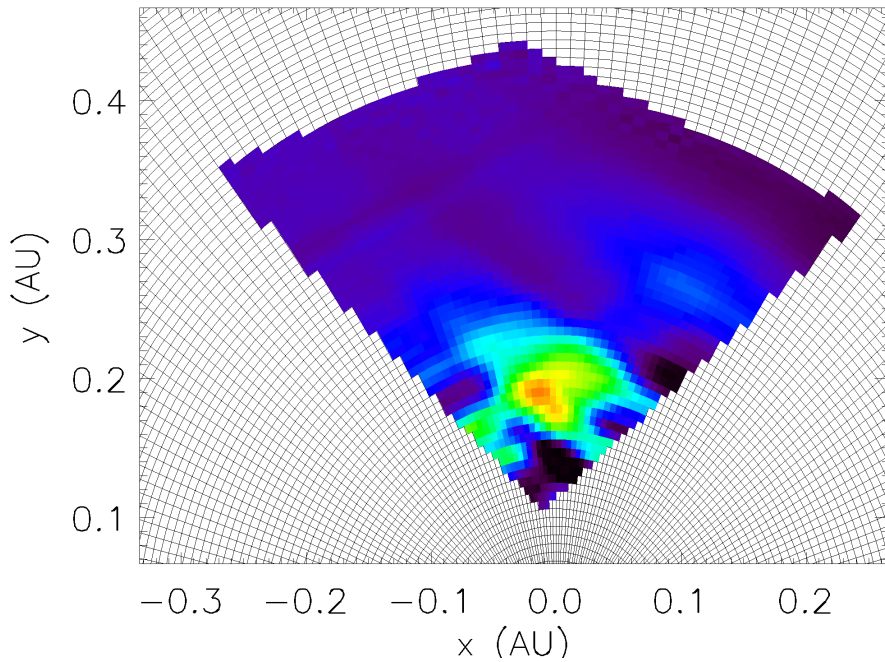
Density (peak) = $31.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $27 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

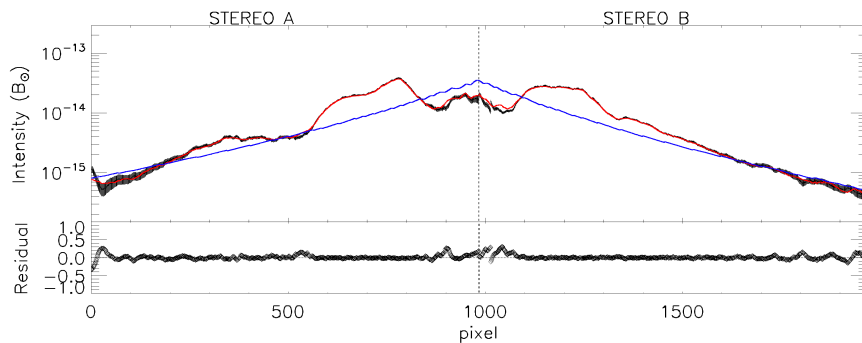
Density Results – Dec 2008 CME



13 Dec 2008 04:09

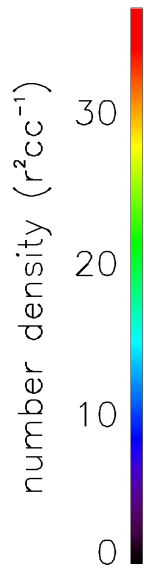
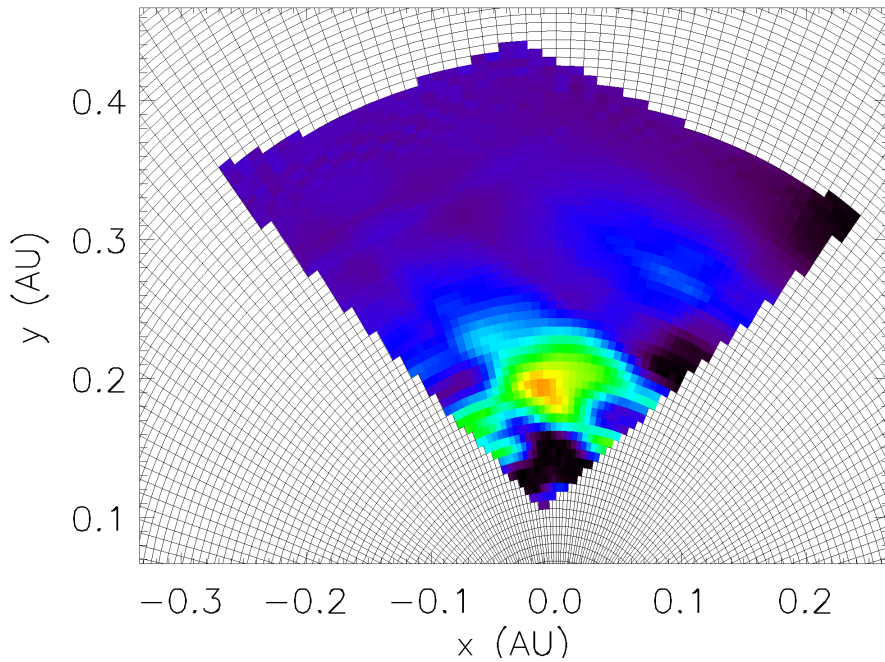
Density (peak) = $31.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $28 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

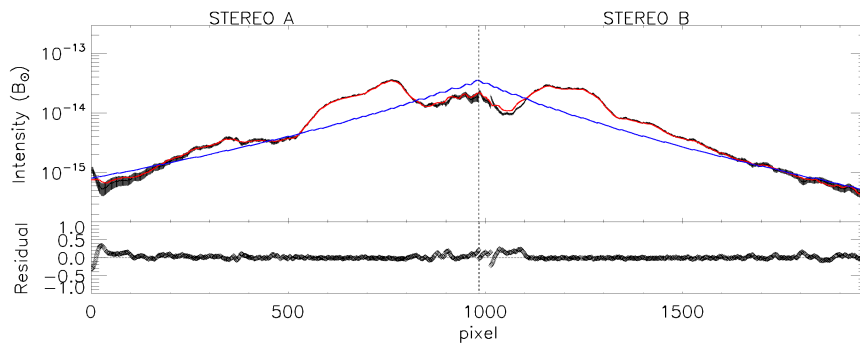
Density Results – Dec 2008 CME



13 Dec 2008 04:49

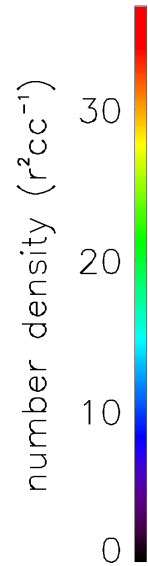
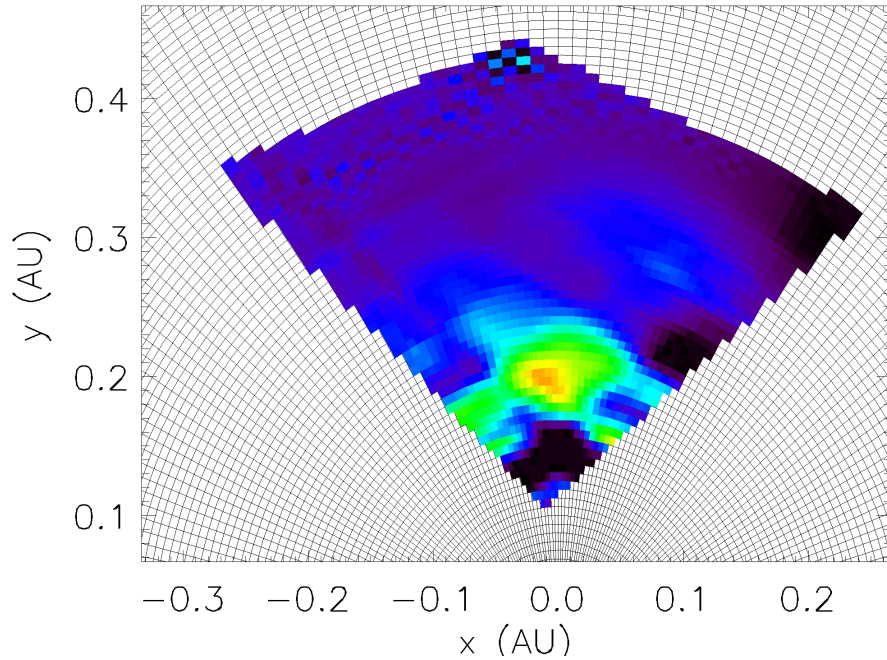
Density (peak) = $30.5 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $28 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

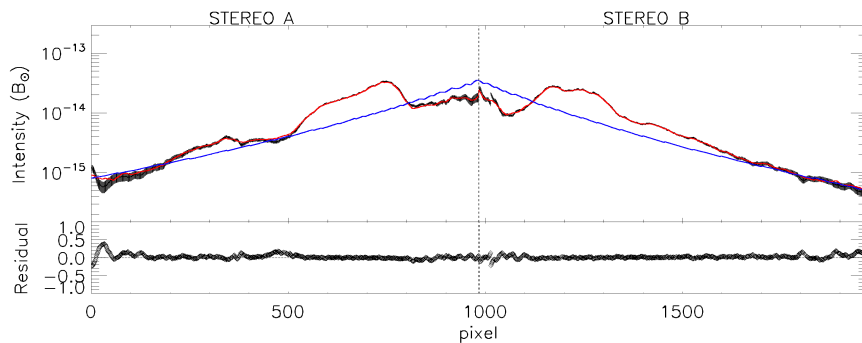
Density Results – Dec 2008 CME



13 Dec 2008 05:29

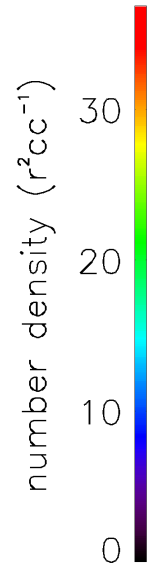
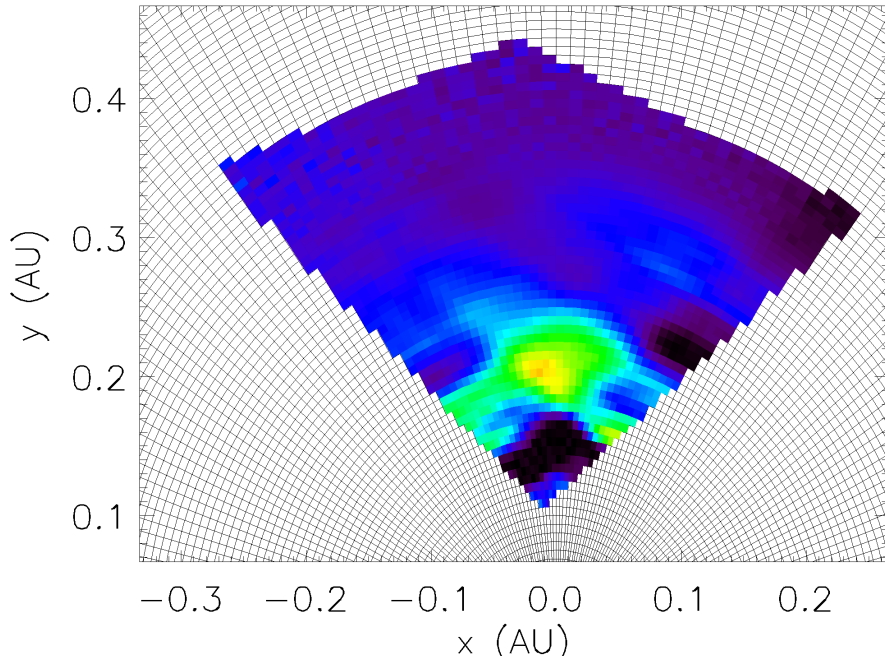
Density (peak) = $30.4 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $29 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

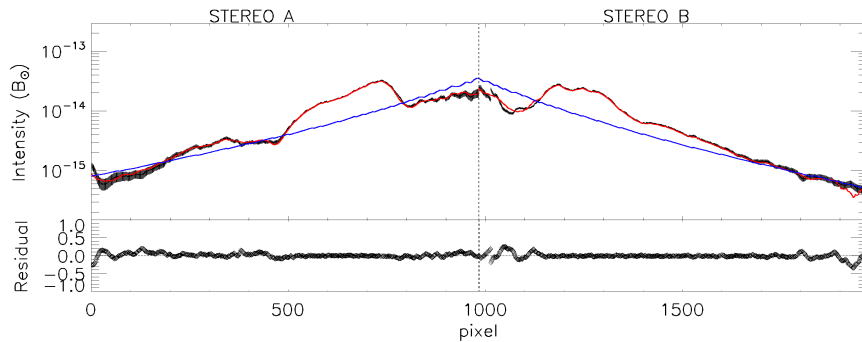
Density Results – Dec 2008 CME



13 Dec 2008 06:09

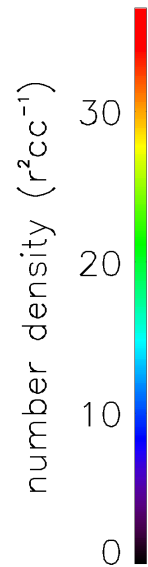
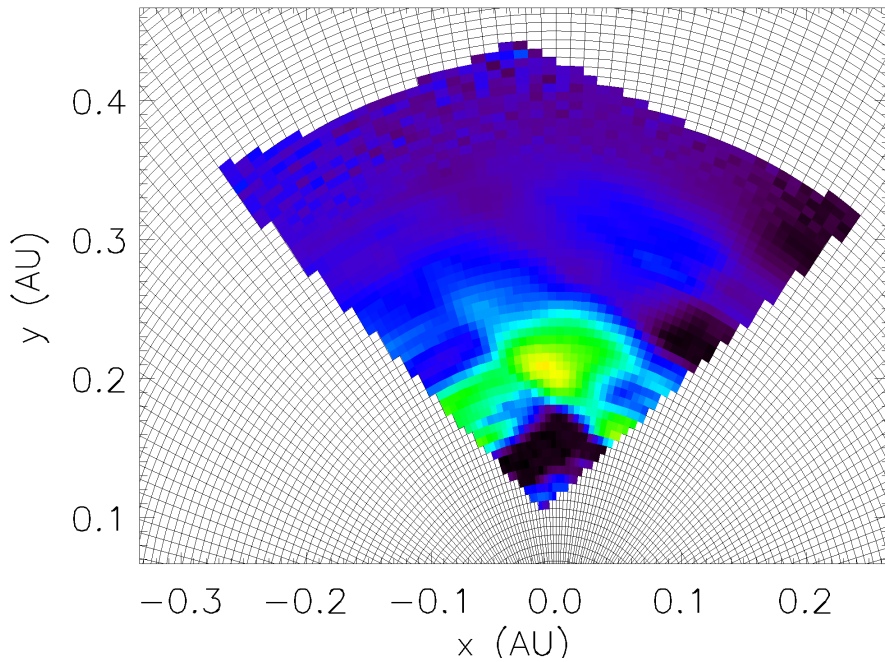
Density (peak) = $29.3 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $30 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

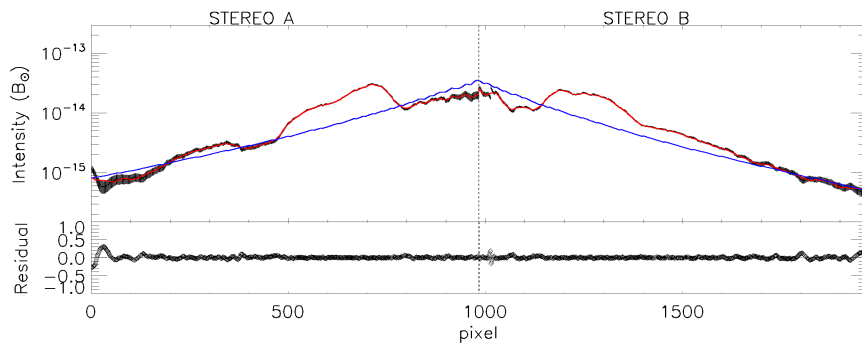
Density Results – Dec 2008 CME



13 Dec 2008 06:49

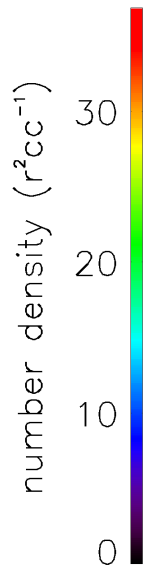
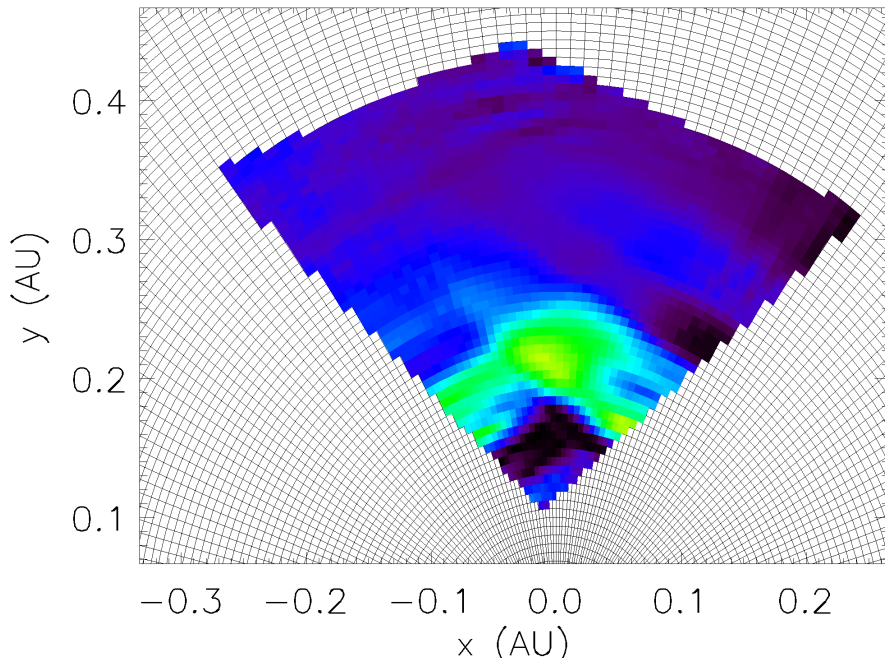
Density (peak) = $28.3 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $31 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

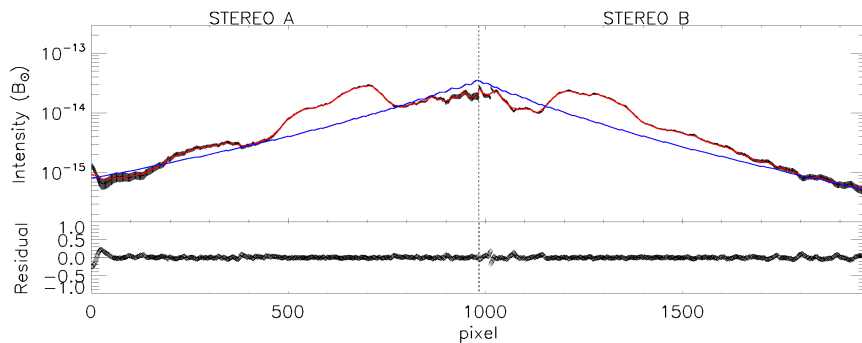
Density Results – Dec 2008 CME



13 Dec 2008 07:29

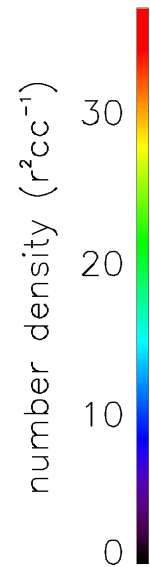
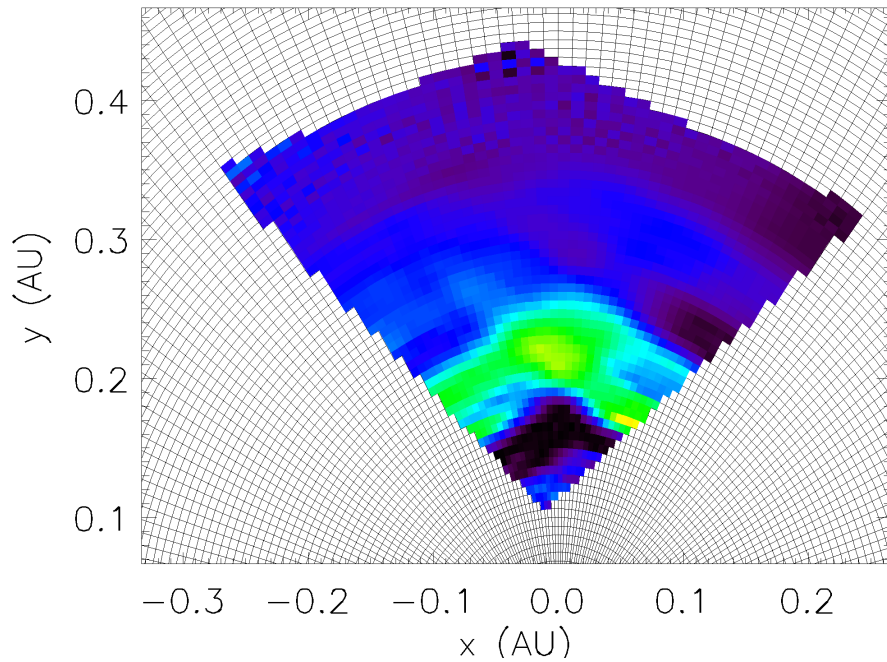
Density (peak) = $26.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $32 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

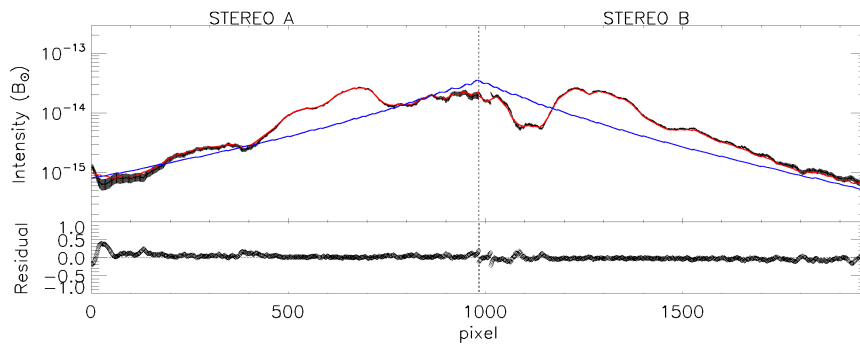
Density Results – Dec 2008 CME



13 Dec 2008 08:09

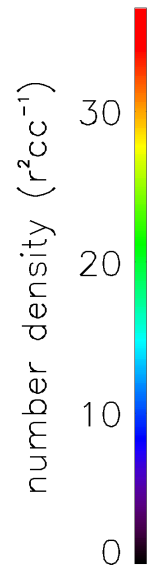
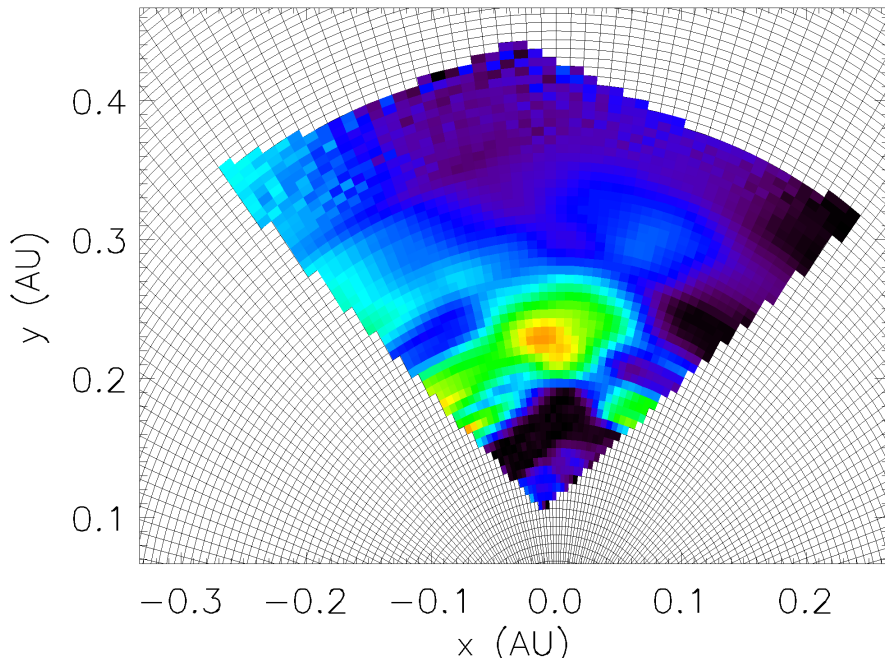
Density (peak) = $26.0 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $33 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

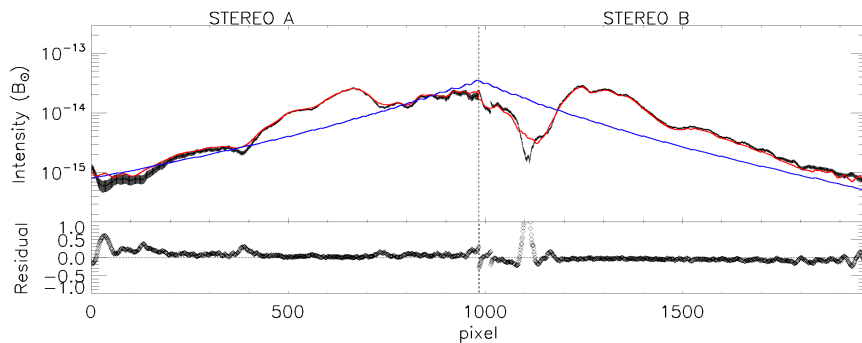
Density Results – Dec 2008 CME



13 Dec 2008 08:49

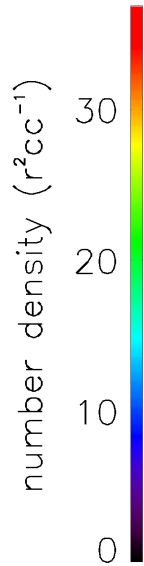
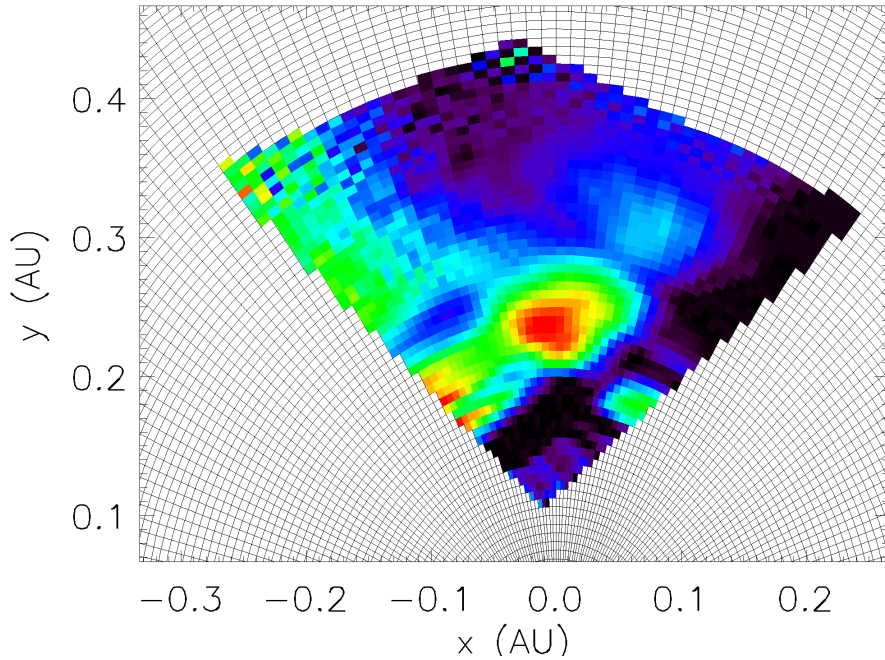
Density (peak) = $30.6 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $34 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

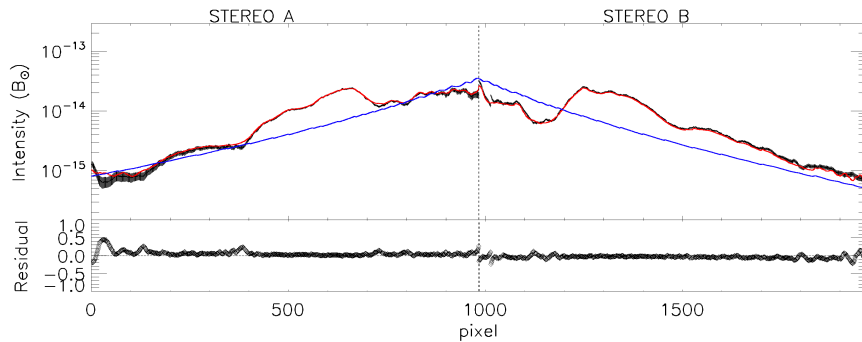
Density Results – Dec 2008 CME



13 Dec 2008 09:29

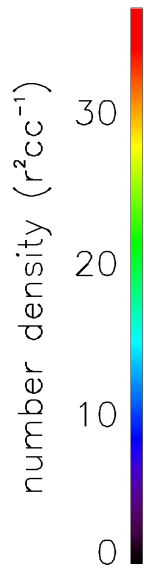
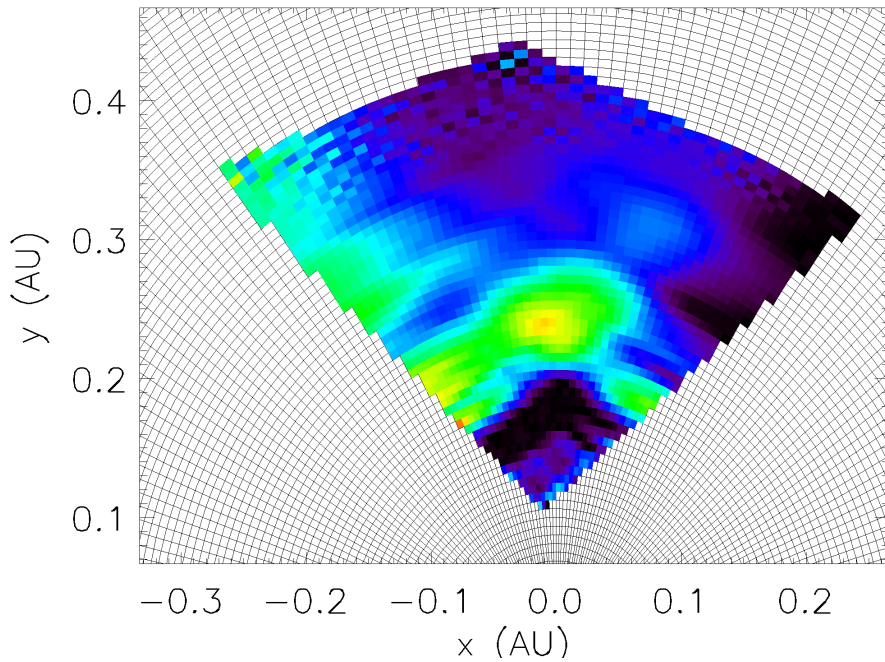
Density (peak) = $35.9 e^- r^2 cc^{-1}$

Distance = $35 * 10^6 km$



- Data
- Initial guess
- Solution

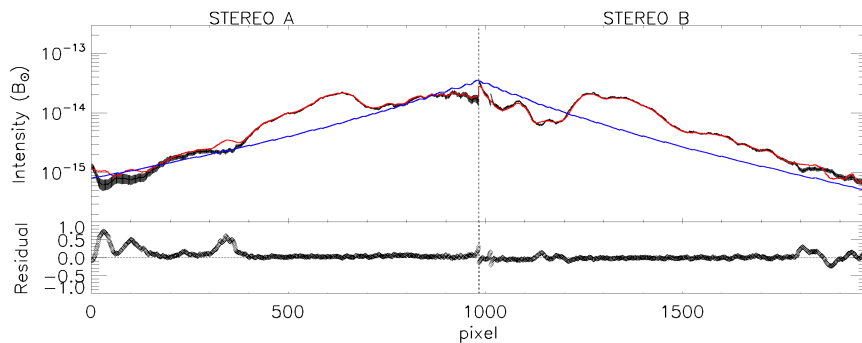
Density Results – Dec 2008 CME



13 Dec 2008 10:09

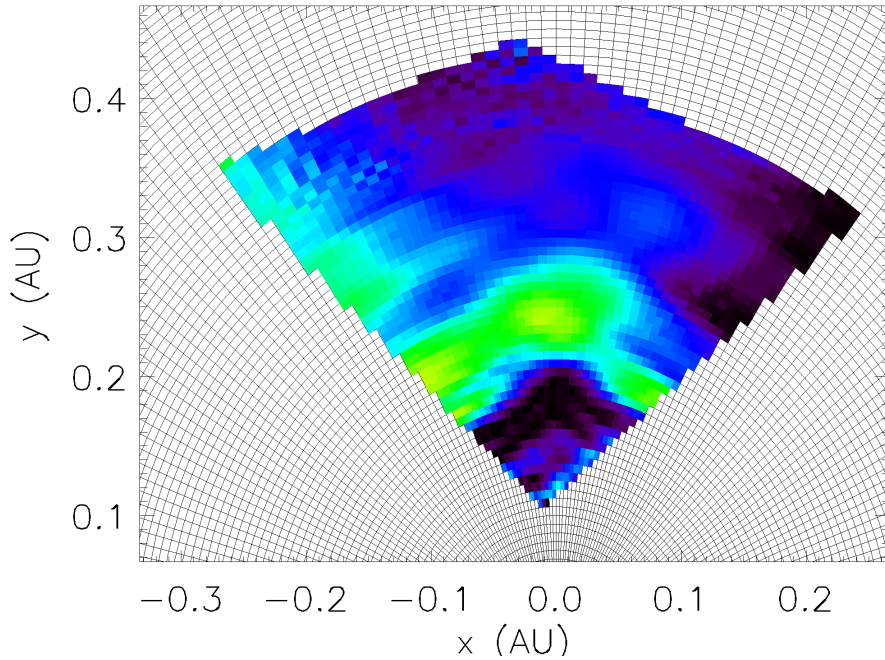
Density (peak) = $29.0 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $36 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

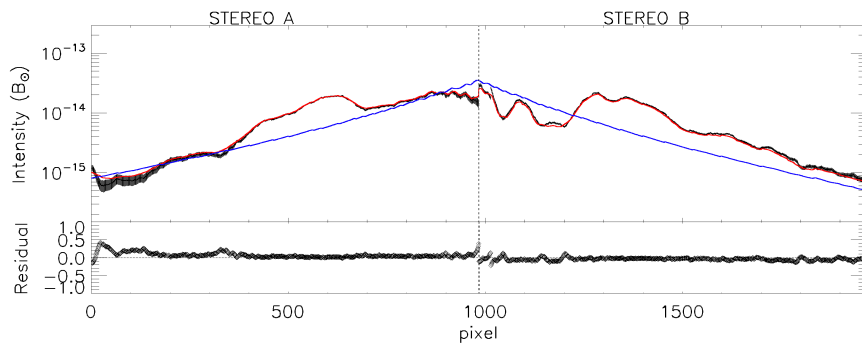
Density Results – Dec 2008 CME



13 Dec 2008 10:49

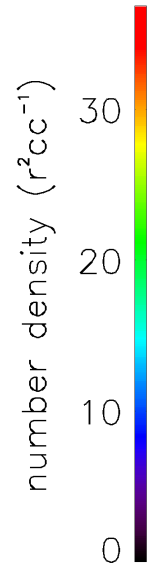
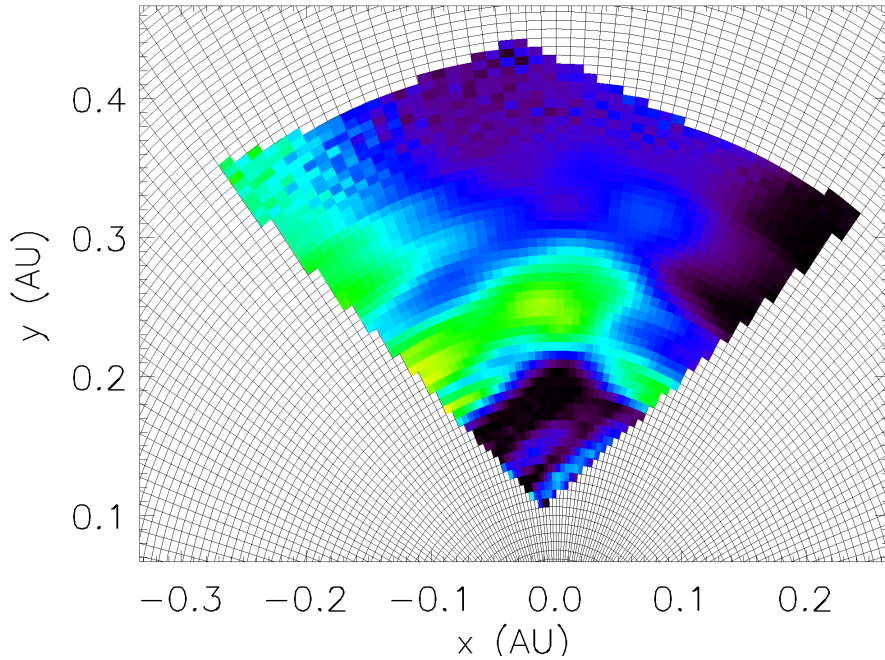
Density (peak) = $25.9 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $36 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

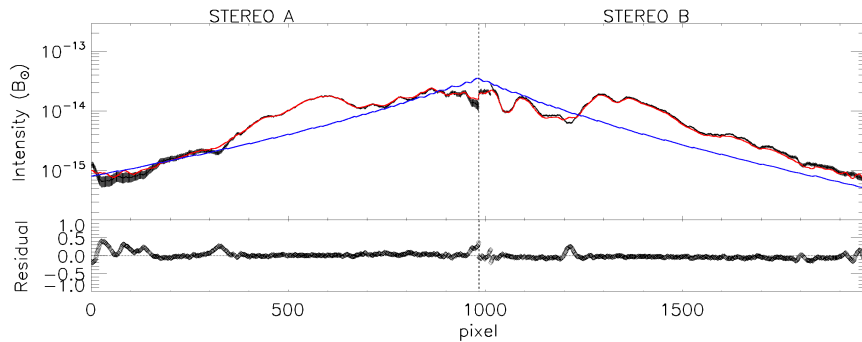
Density Results – Dec 2008 CME



13 Dec 2008 11:29

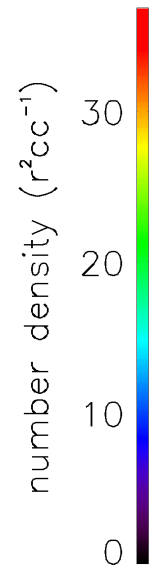
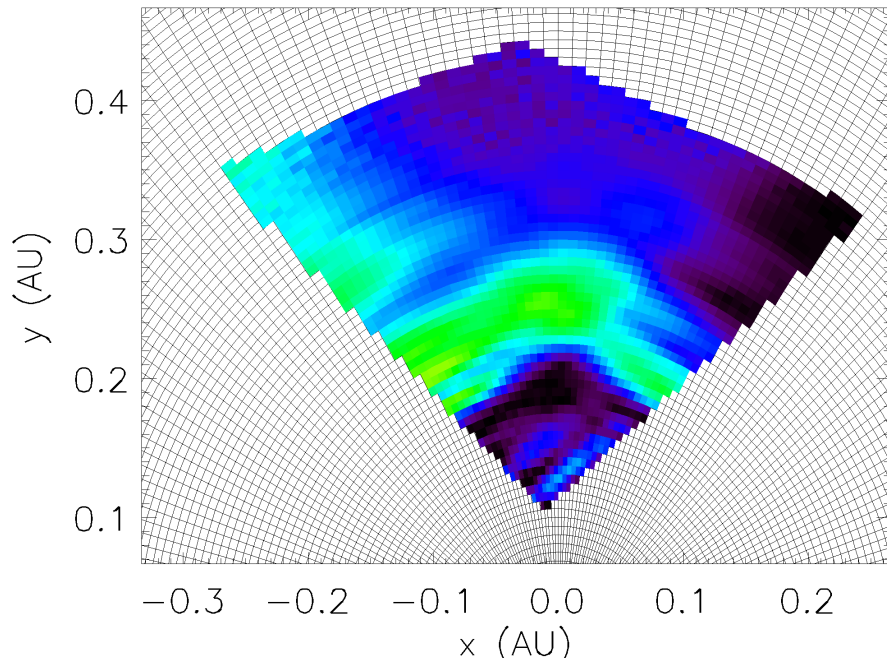
Density (peak) = $25.6 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $38 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

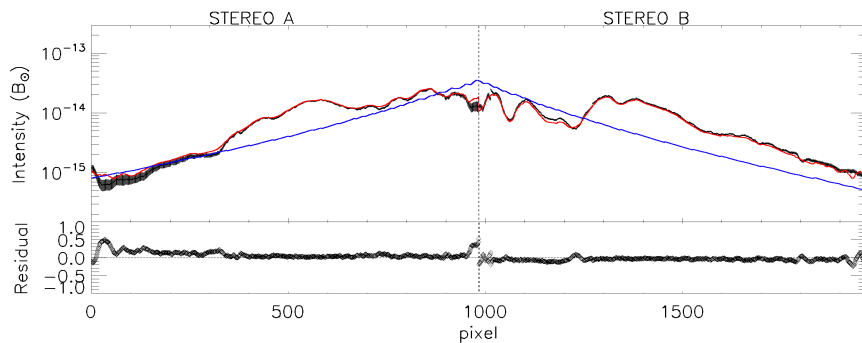
Density Results – Dec 2008 CME



13 Dec 2008 12:09

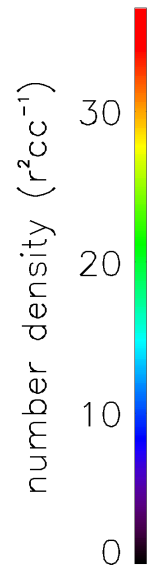
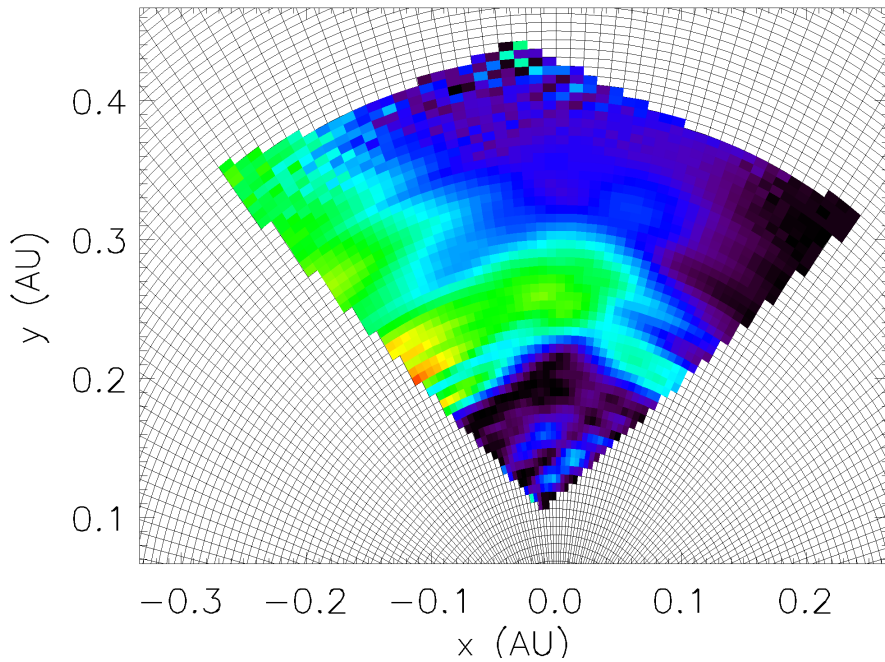
Density (peak) = $23.4 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $38 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

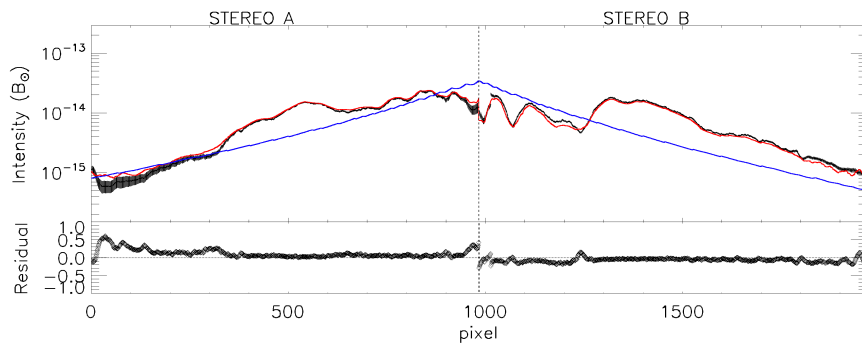
Density Results – Dec 2008 CME



13 Dec 2008 12:49

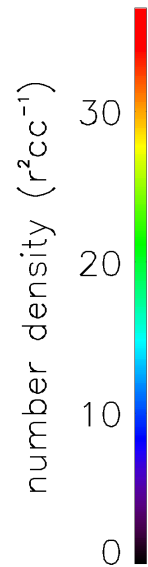
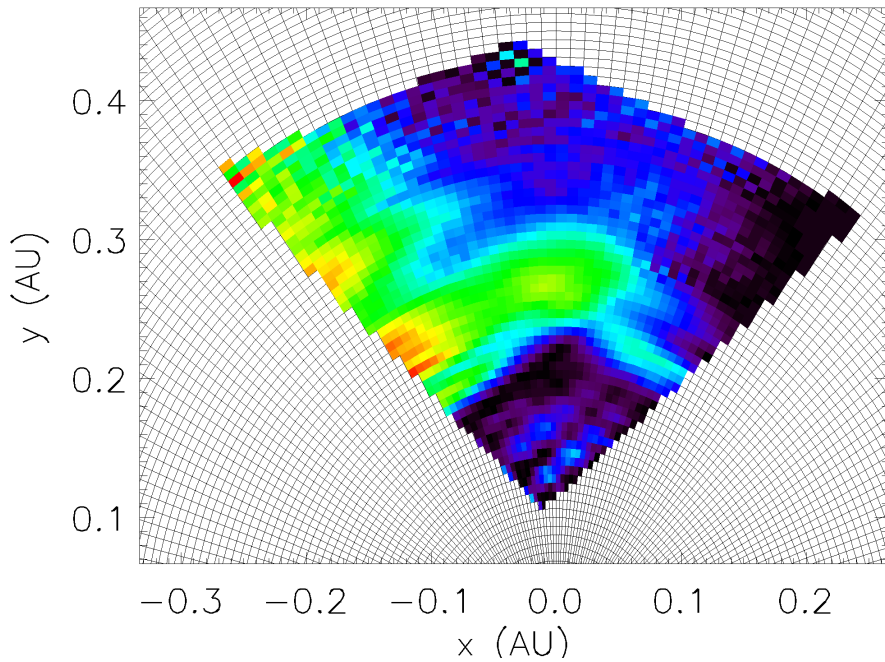
Density (peak) = $24.5 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $38 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

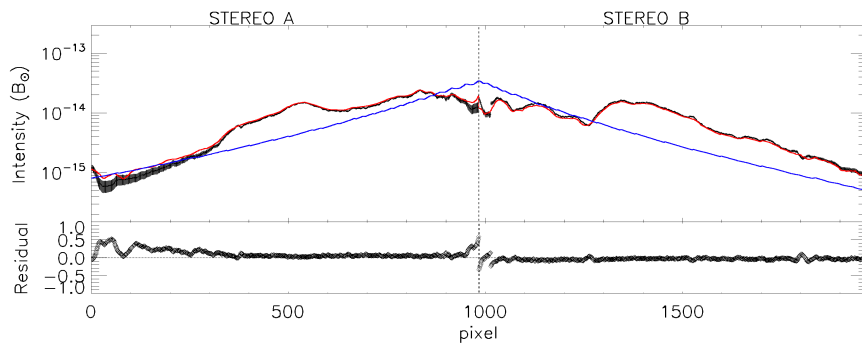
Density Results – Dec 2008 CME



13 Dec 2008 13:29

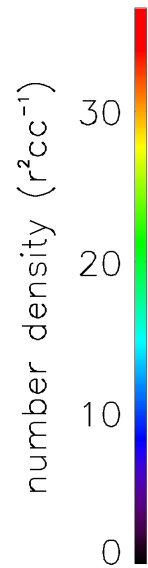
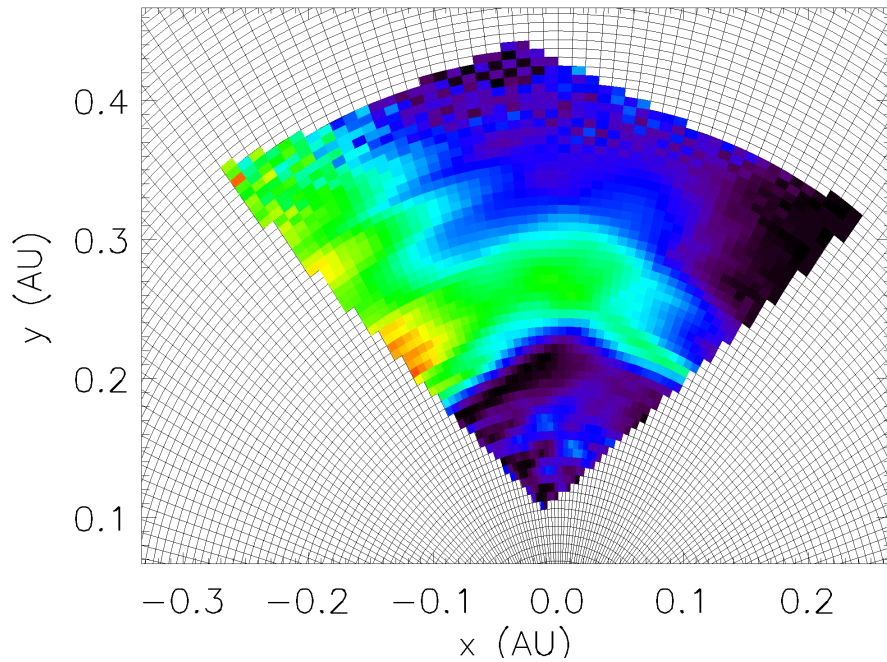
Density (peak) = $25.9 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $39 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

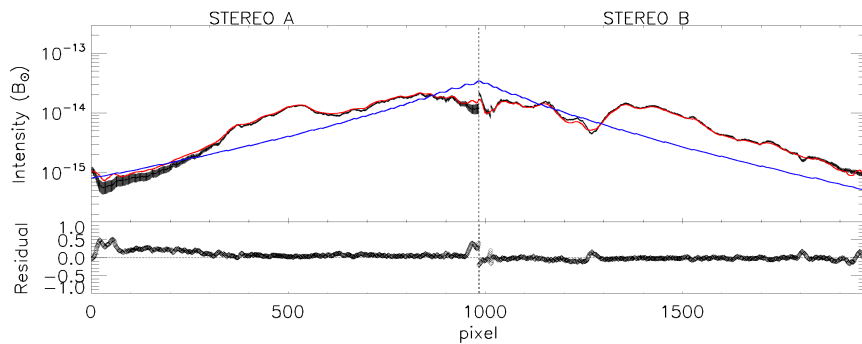
Density Results – Dec 2008 CME



13 Dec 2008 14:09

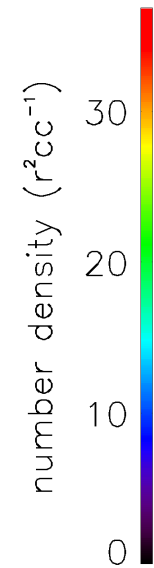
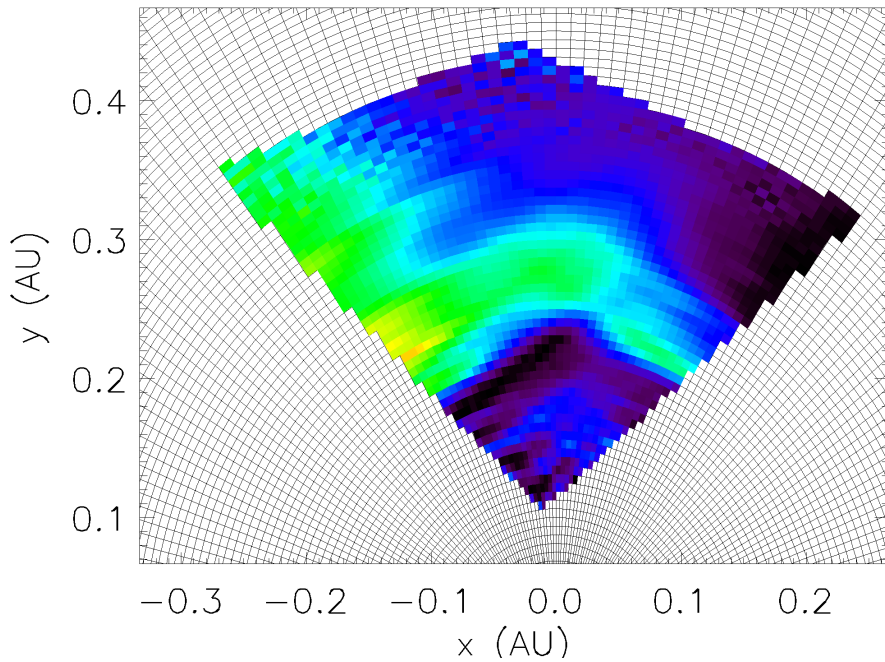
Density (peak) = $22.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $40 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

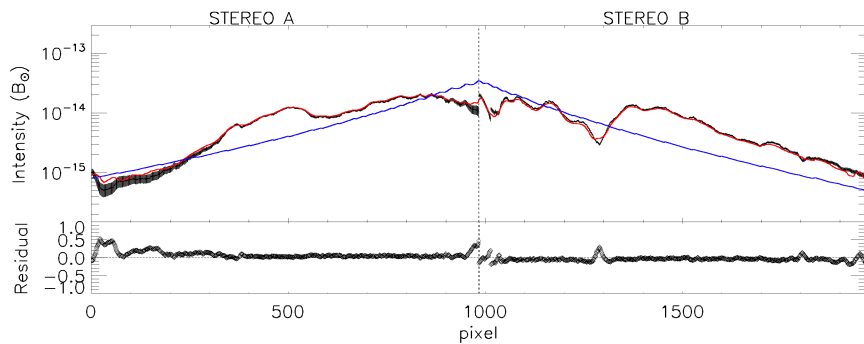
Density Results – Dec 2008 CME



13 Dec 2008 14:49

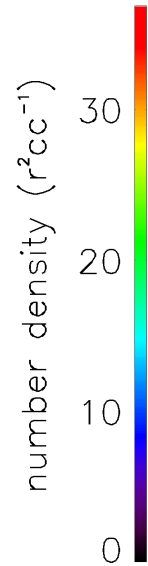
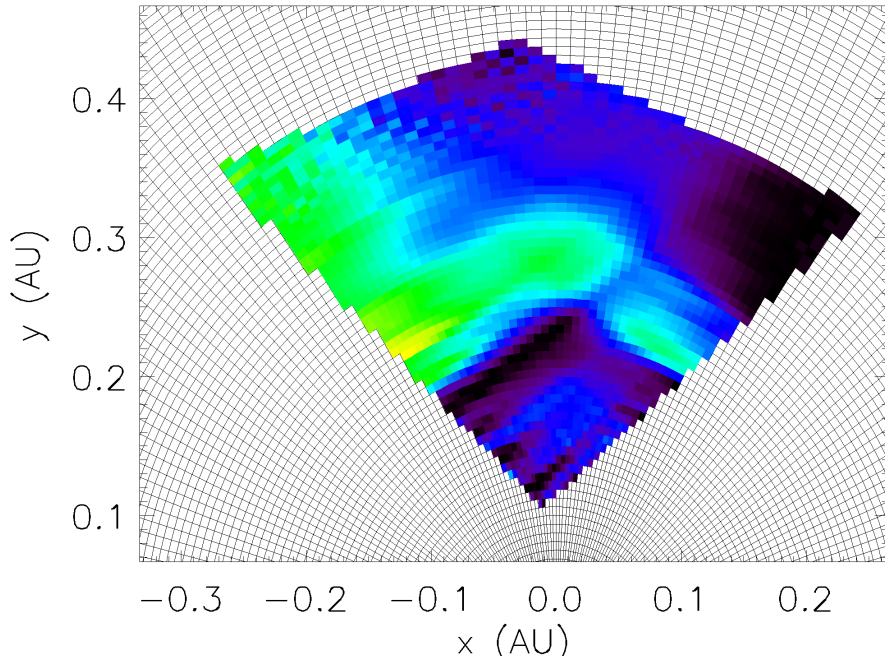
Density (peak) = $20.3 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $41 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

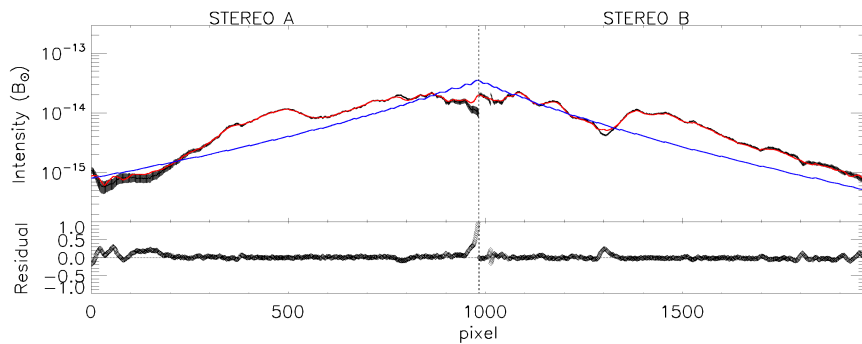
Density Results – Dec 2008 CME



13 Dec 2008 15:29

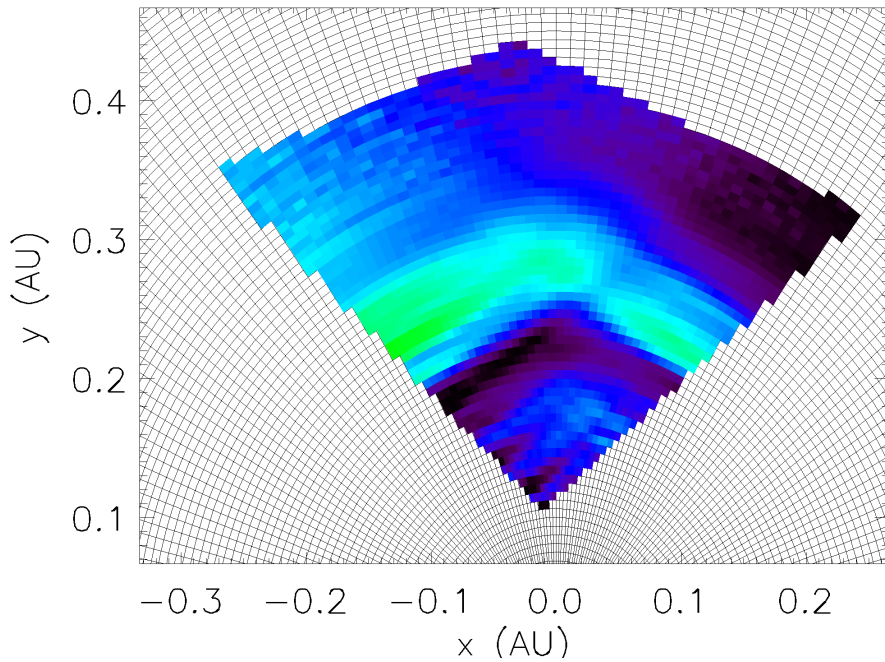
Density (peak) = $20.4 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $42 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

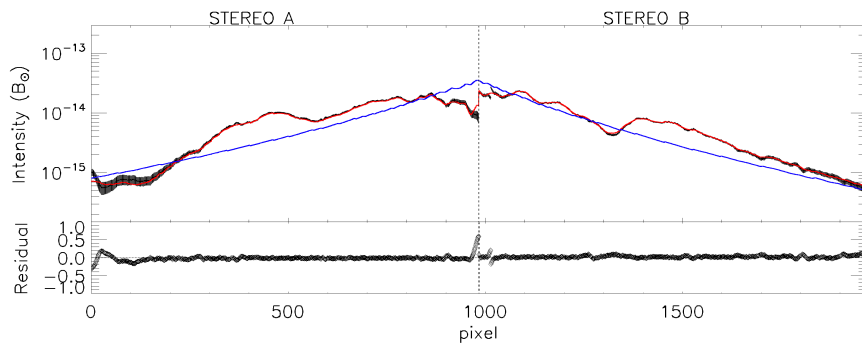
Density Results – Dec 2008 CME



13 Dec 2008 16:09

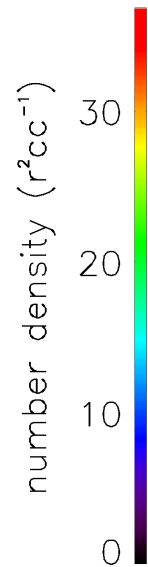
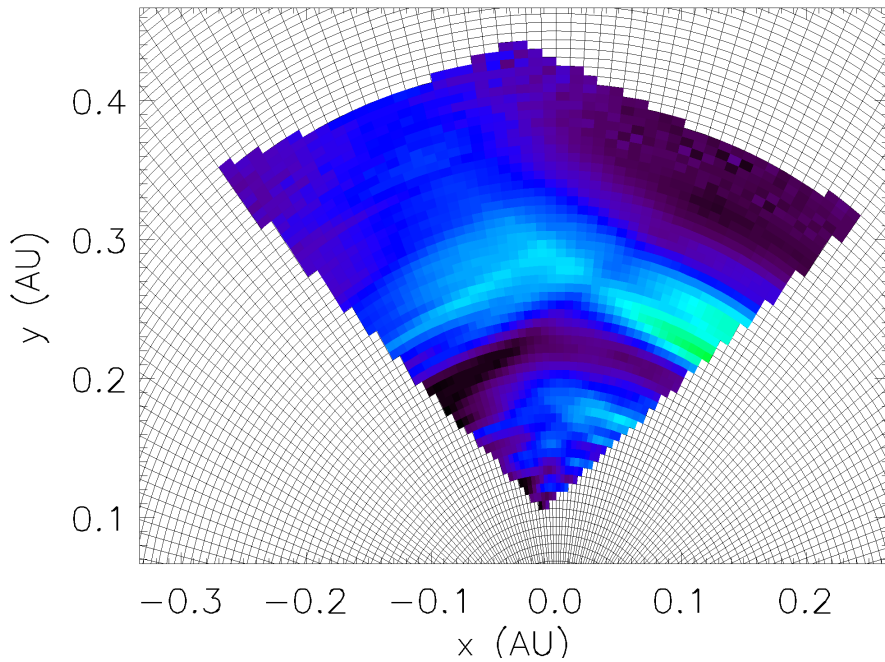
Density (peak) = $17.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $42 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

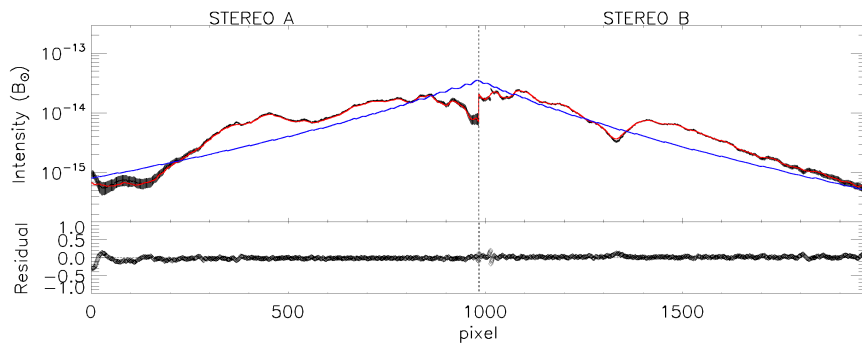
Density Results – Dec 2008 CME



13 Dec 2008 16:49

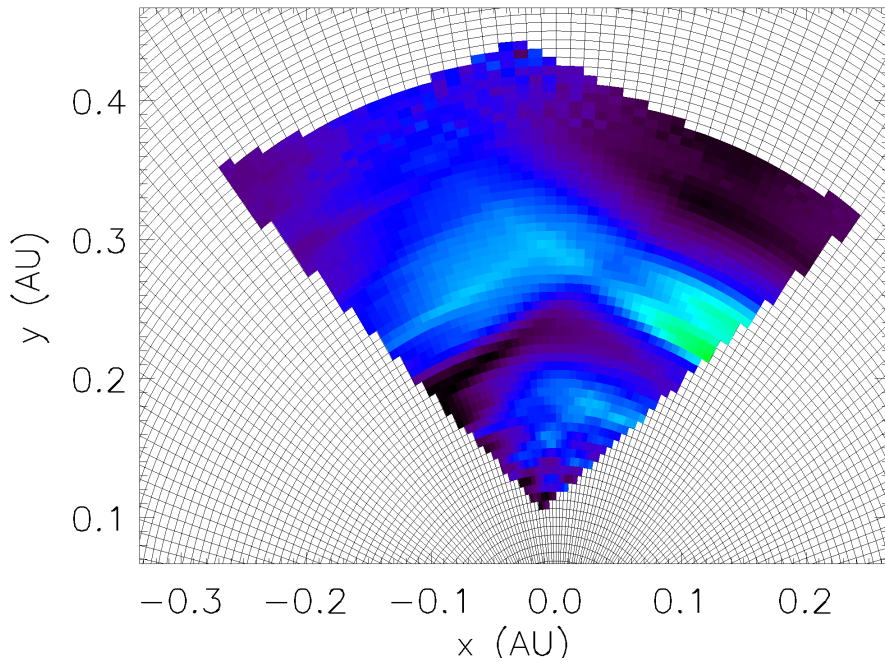
Density (peak) = $14.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $41 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

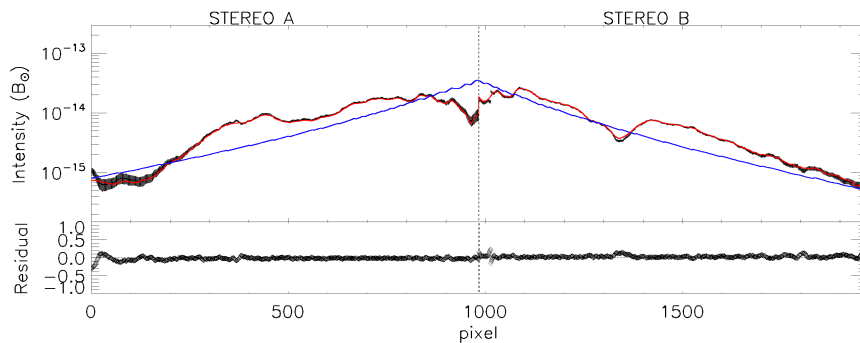
Density Results – Dec 2008 CME



13 Dec 2008 17:29

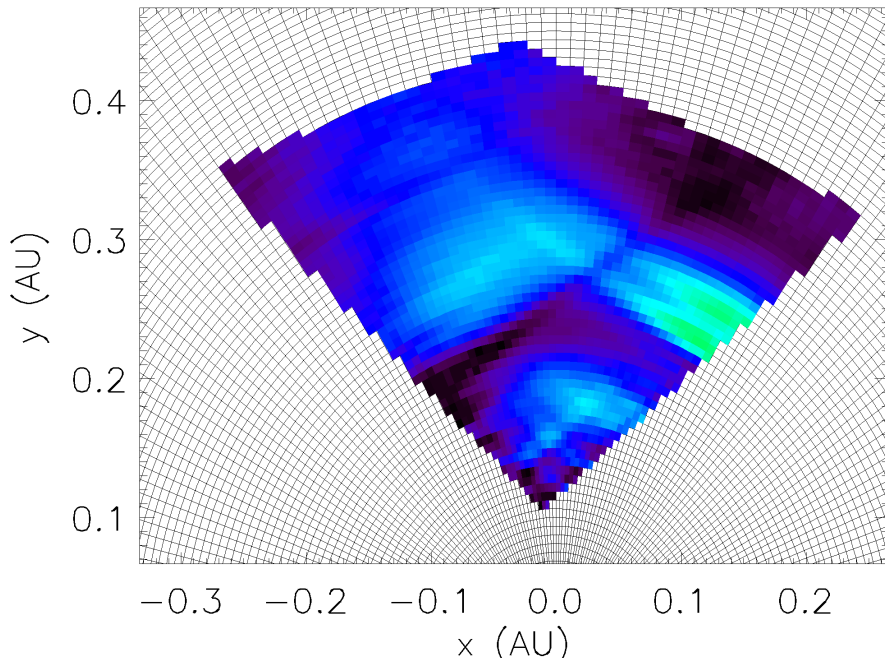
Density (peak) = $13.6 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $44 \times 10^6 \text{ km}$



- Data
- Initial guess
- Solution

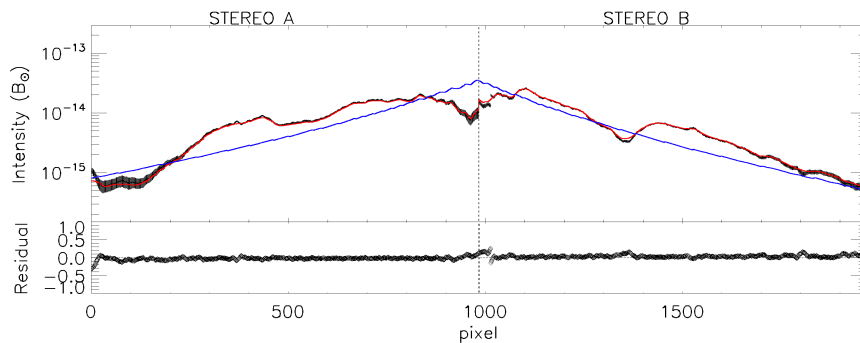
Density Results – Dec 2008 CME



13 Dec 2008 18:09

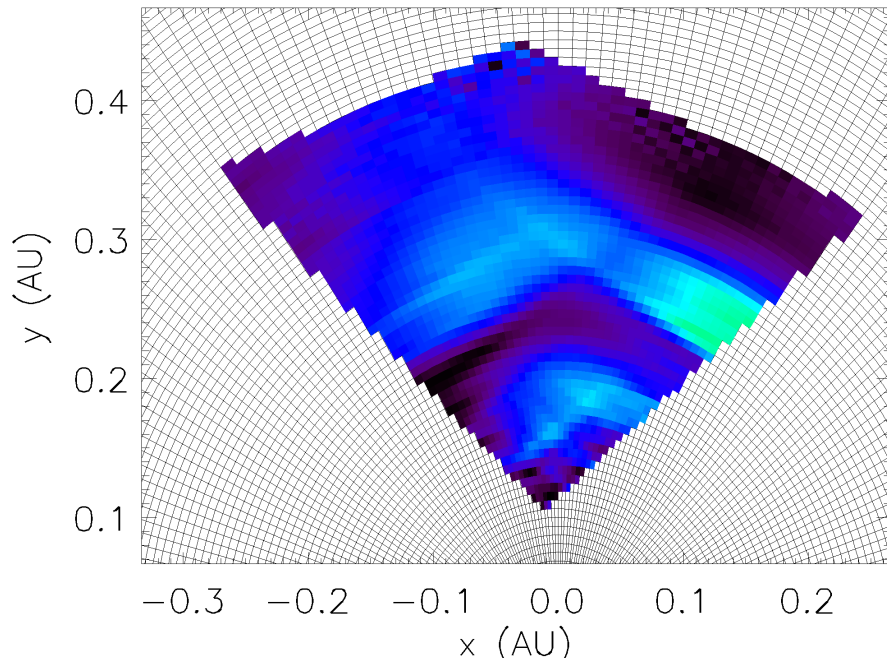
Density (peak) = $14.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $45 \cdot 10^6 \text{ km}$



- Data
- Initial guess
- Solution

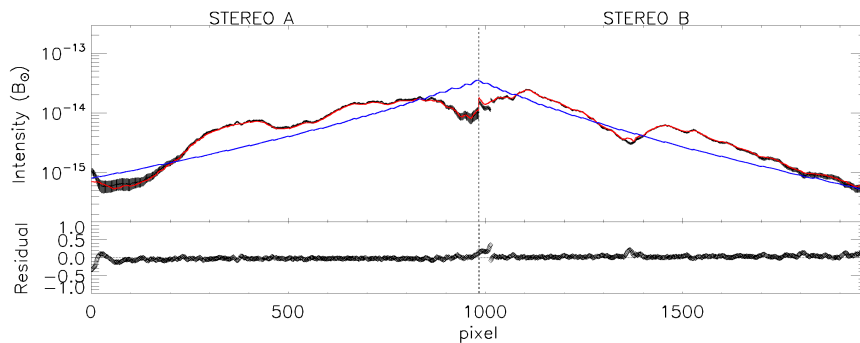
Density Results – Dec 2008 CME



13 Dec 2008 18:49

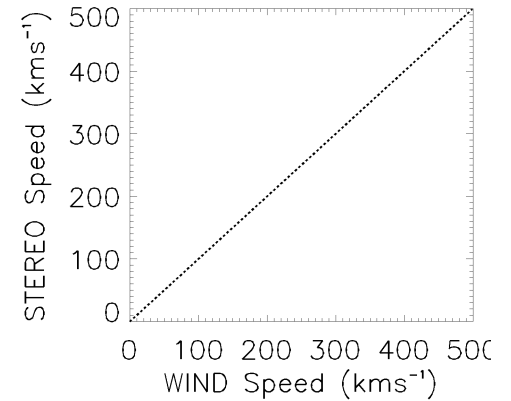
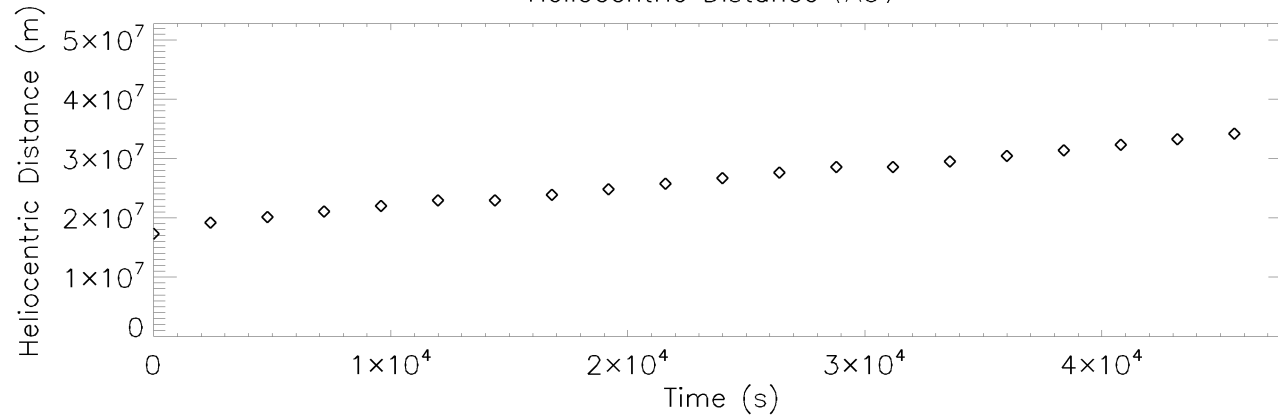
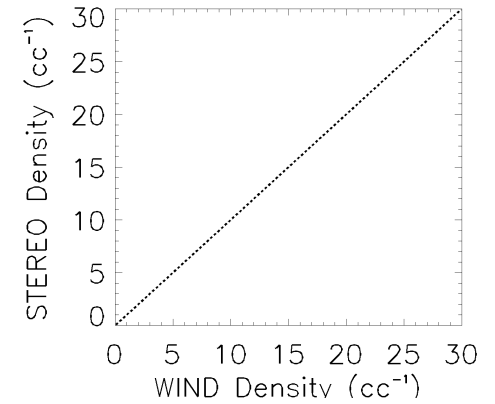
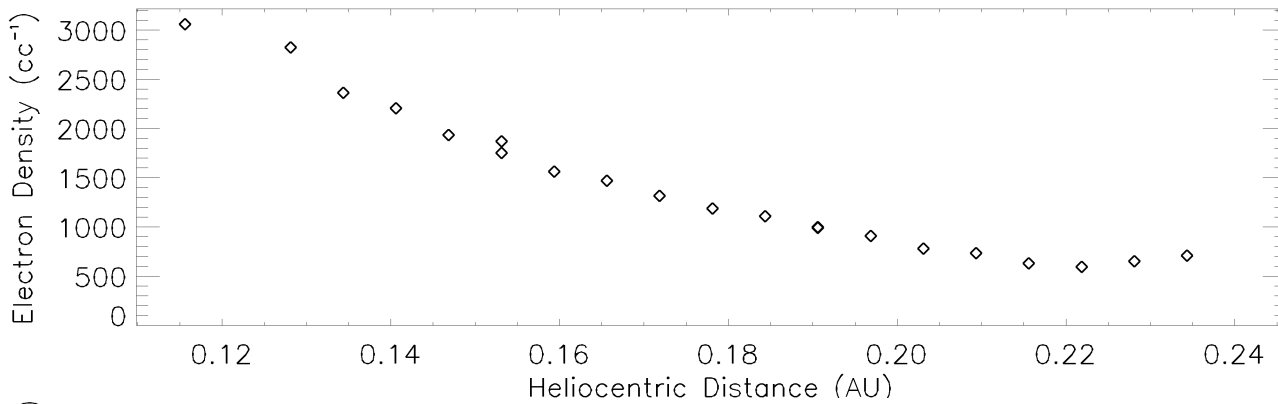
Density (peak) = $13.1 \text{ e}^- r^2 \text{ cc}^{-1}$

Distance = $44 \times 10^6 \text{ km}$

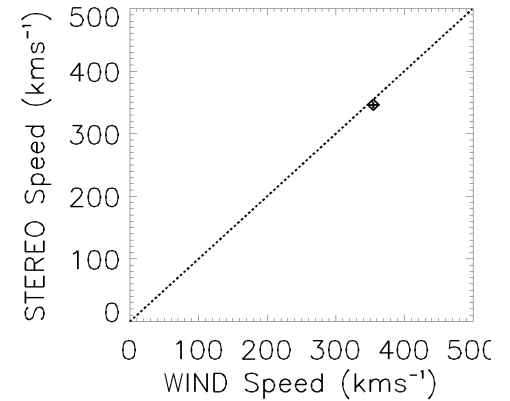
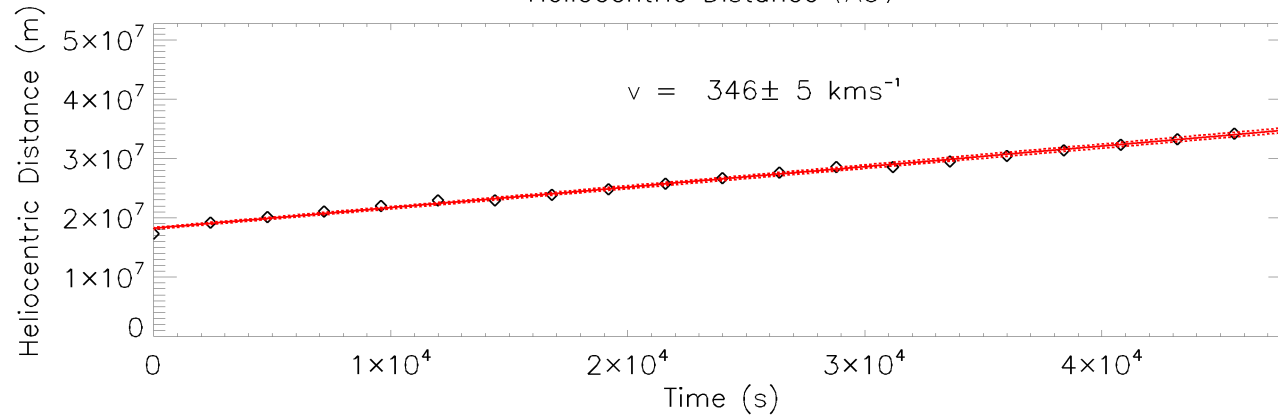
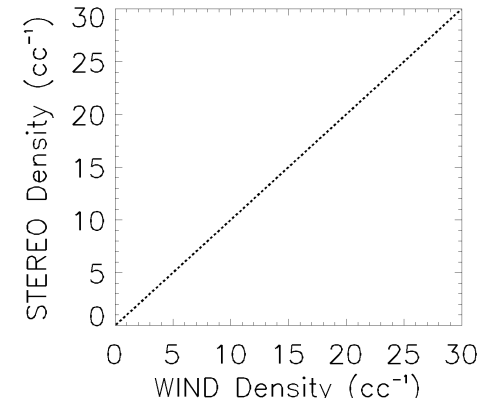
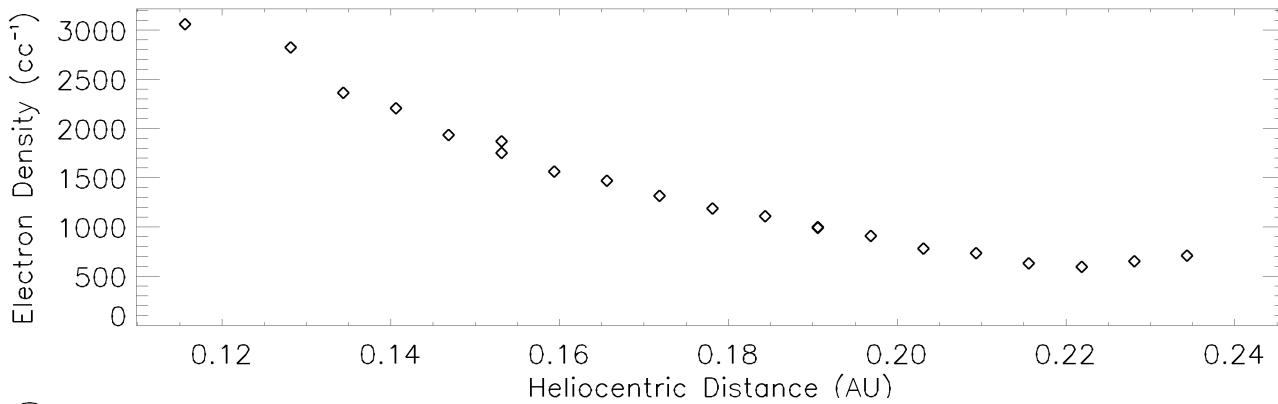


- Data
- Initial guess
- Solution

Dec 2008 CME Speed and Density

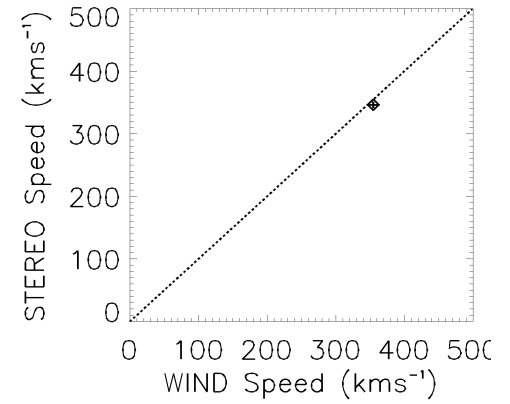
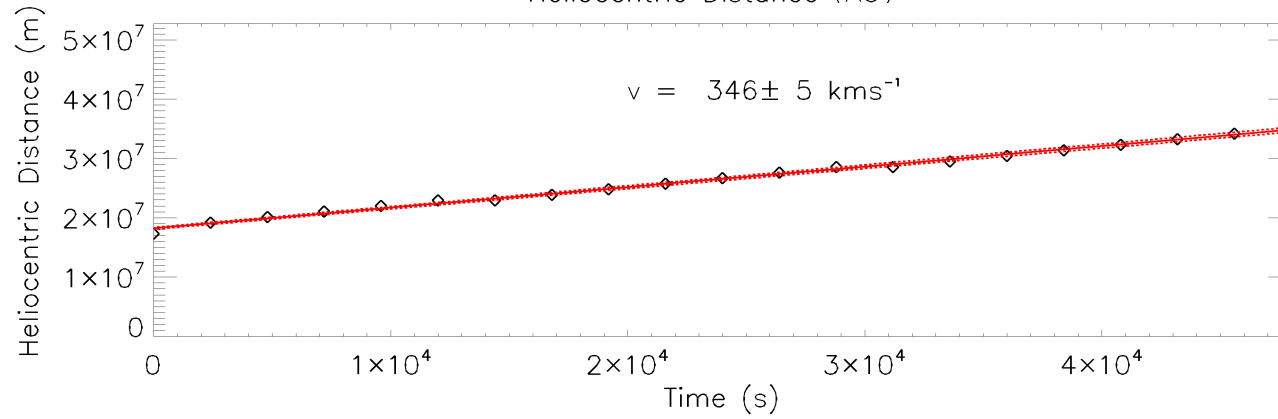
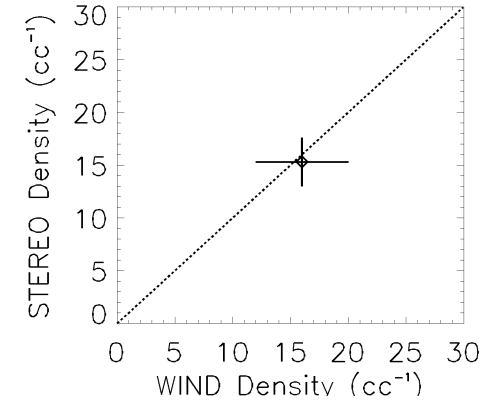
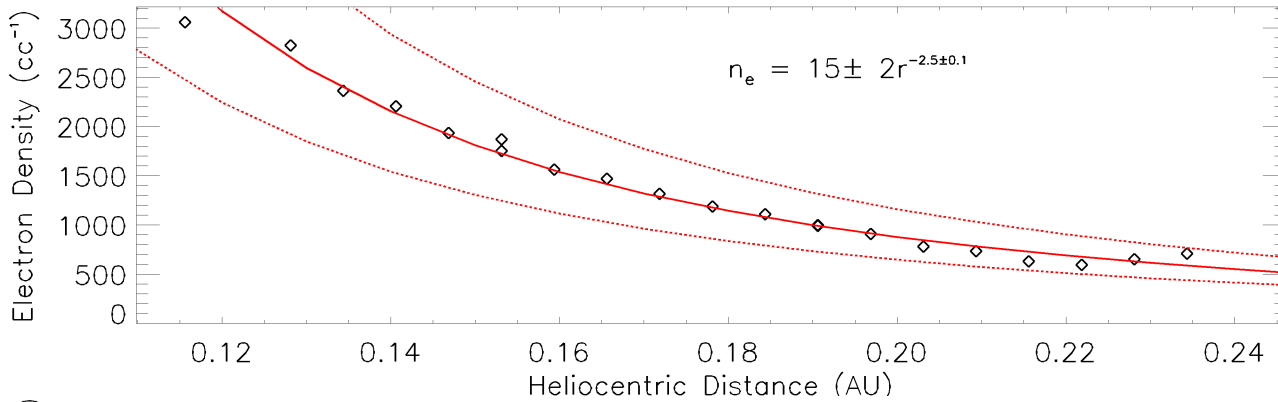


Dec 2008 CME Speed and Density



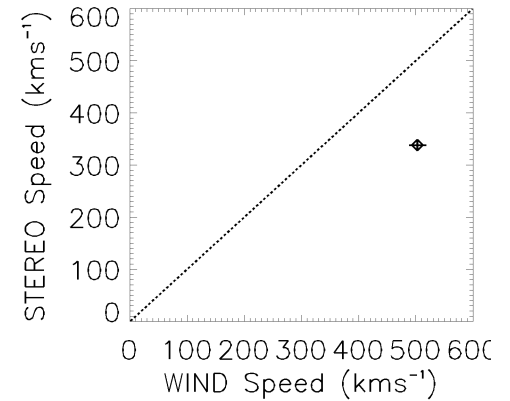
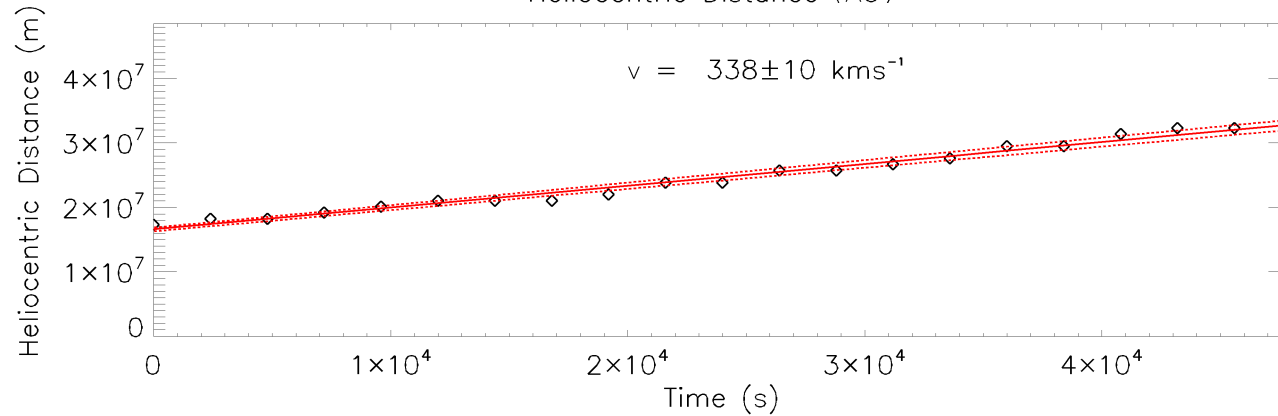
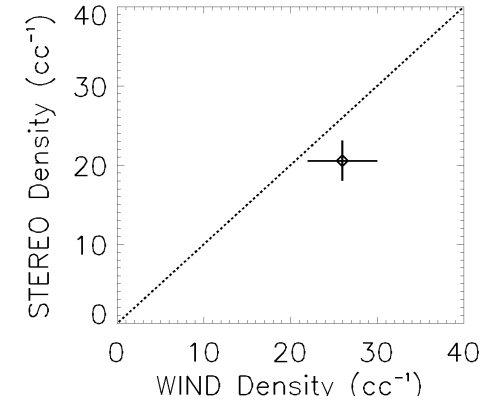
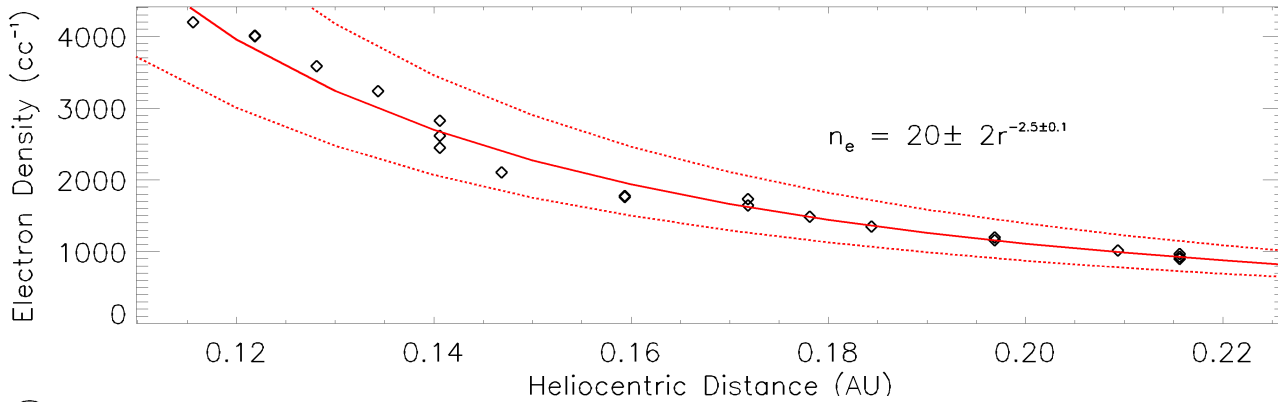
- Assume constant velocity

Dec 2008 CME Speed and Density

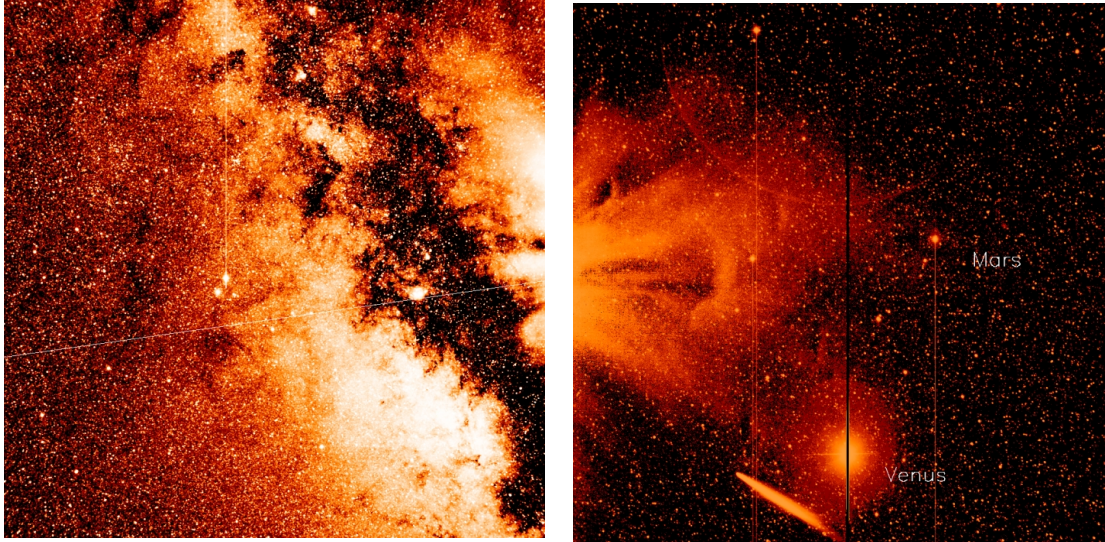


- Assume constant velocity
- Assume density and radius follow power law
- Regularisation terms can be tuned to give more accurate value for density

Oct 2011 CME Speed and Density



Limitations



- The galactic plane passes through the FOV of each spacecraft ~ 2 times per year
- Planets and comets frequently obscure images
- Relative position of the two spacecraft causes line-of-sight problems
- CME must be Earth-directed

Summary

- Electron density distribution in the ecliptic has been measured using combined HI data from both STEREO spacecraft
- The technique may be used if corrections are applied to produce a physically consistent K-corona intensity
- Iterative algorithm typically produces a density distribution with a mean residual of less than 5% of observations
- May be used to estimate CME speeds and densities
- Only two CMEs to which this technique has been applied

